CLASS X Real numbers:-

EUCLIDS Division Lemma and Algorithm

Euclid's division lemma tells us about divisibility of integers. It states that any positive integer "**a'** can be divided by any other positive integer **'b'** in a such a way that it leaves a remainder r (where r<b), We may recognize this as the usual long division process ;

Euclid's division lemma provides a stepwise procedure to compute the HCF of any two +ve integers which is known as Euclid's division algorithm.

Euclid's division lemma : states that, for any two positive integers say **a and b ,** there exists two unique whole numbers say **q and r**, such that

a=bq + r , where 0 ≤ r < b

Here a = Dividend , b=Divisor , q=Quotient and r = Reminder
It can be written as
Dividend= (Divisor ×Quotient) +Reminder
Note : A Lemma is a proven statement used for proving another statement.
Euclid's Division Algorithm :
Euclid's division algorithm is a technique to complete the highest common factor (HCF) of two or three given +ve integers.

According to this, the HCF of any two positive integers a and b with a> b is obtained as follows.

Step I Apply the division lemma to find q and r where a≠ bq + r, 0≤ r < b.

Step II If r=0 , the HCF is b If r≠0 apply Euclid's lemma to b and r .

Step III Continue the process till the remainder is zero .The divisor at this stage will be HCF (a,b)

Methods of Finding HCF of three numbers by using Euclid's Division Algorithm

Example : Find the HCF of 180, 252 and 324 by using Euclid's Division Lemma.

Step I Arrange the given three numbers be a, b, c such that a>b>c .

Given numbers are 180 , 252 and 324.

324 > 252 > 180.

Step II HCF of two numbers a and b by EDA say d
Now on applying EDL for 324 and 252 . we get
324 = (252 x 1) +72
Here 72 ≠ 0
So again apply EDL with dividend 252 and new divisor 72, we get
252= 72 x 3 + 36

Here remainder = 36 ≠ 0

So again applying EDL with new dividend 72 and new divisor 36, we get
72 = 36 x 2 + 0.
Here R = 0.
So HCF of 324 and 252 is 36.

Step III:- Now , find the HCF of two numbers (remaining numbers) and d
(HCF of a and b by EDA.
So applying EDL for 180 and 36 .
We get 180 = 36 ×5 +0
R=0
So HCF of 180 and 36 is 36
Step IV The HCF obtained in step 3 is required HCF of given three numbers
Hence HCF of 324 , 252 and 180 is 36.

Fundamental Theorem of Arithmetic Prime and Composite numbers'

A number is called a prime number if it has no factor other than 1 and the number itself
e.g. 2,3,5,7,13,19…. are prime numbers.
A number is called a composite number if it has at least one factor other than 1 and itself .
e.g. 4,6,8,10,…. 188 are composite numbers.

Note:- 1 is neither prime nor composite.
2, is the smallest prime number . It is the only even number.
The smallest composite even number is 4 and smallest composite odd number is 9.
Prime number have only two factors 1 and itself.
Factor of a Number
If a number divides another number exactly (without leaving any remainder), than the number which divides is called a factor of the number that has been divided.

e.g 1,2,3,4,6,12 are the factors of 24.

Fundamental Theorem Of Arithmetic

According to the Fundamental theorem of arithmetic ,every composite number can be written (factorized) as the product of primes and this factorization is unique , apart from the order in which the prime factors occur.

Fundamental Theorem of Arithmetic is also called Unique factorization Theorem.

Composite number = Product of prime numbers.

OR

An integer greater than 1 , either be a prime no or can be written as a product of prime factors.

HCF and LCM
By Prime Factorization Method:

The method of finding HCF and LCM of two or more positive numbers by using Fundamental theorem of arithmetic is called prime factorization method .In this method we first express the given two or more no's as the product of prime factors separately ,then

1. HCF of two or more numbers = Product of the smallest power of each common prime factor involved in the numbers.
2. LCM of two or more numbers = Products of the greatest power of each prime factor involved in the numbers , with highest power.

Relation between numbers and their HCF and LCM .

1. For any two +ve integers a and b, the relation between these numbers and their HCF and LCM is

HCF (a ,b) ×LCM (a ,b) = a × b

$$\text{HCF}(a, b) = \frac{a \times b}{\text{LCM}(a, b)}$$

Or

$$\text{LCM}(a, b) = a \times b \frac{a \times b}{\text{HCF}(a, b)}.$$

2. For any three positive integers a , b and c , the relation between these numbers and their HCF and LCM is

$$\text{HCF}(a, b, c) = \frac{a \times b \times c \times \text{LCM}(a, b, c)}{\text{LCM}(a, b) \times \text{LCM}(b, c) \times \text{LCM}(c, a)}$$

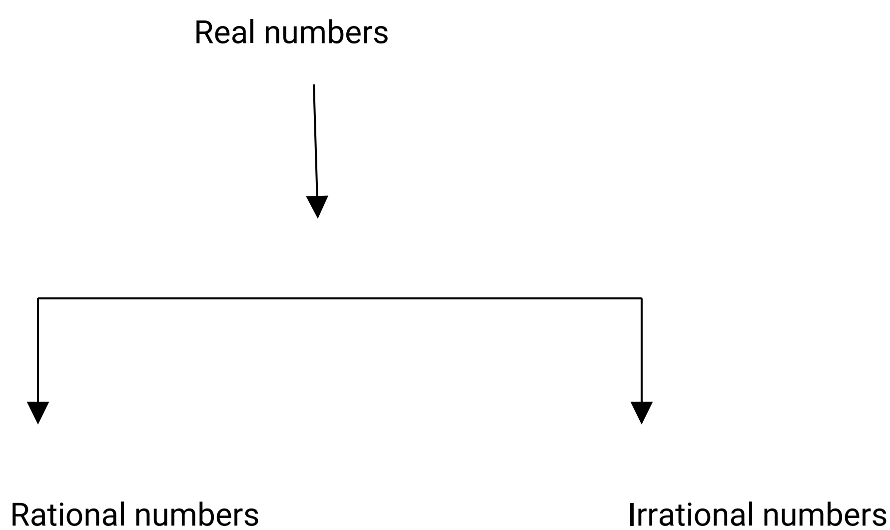
Or

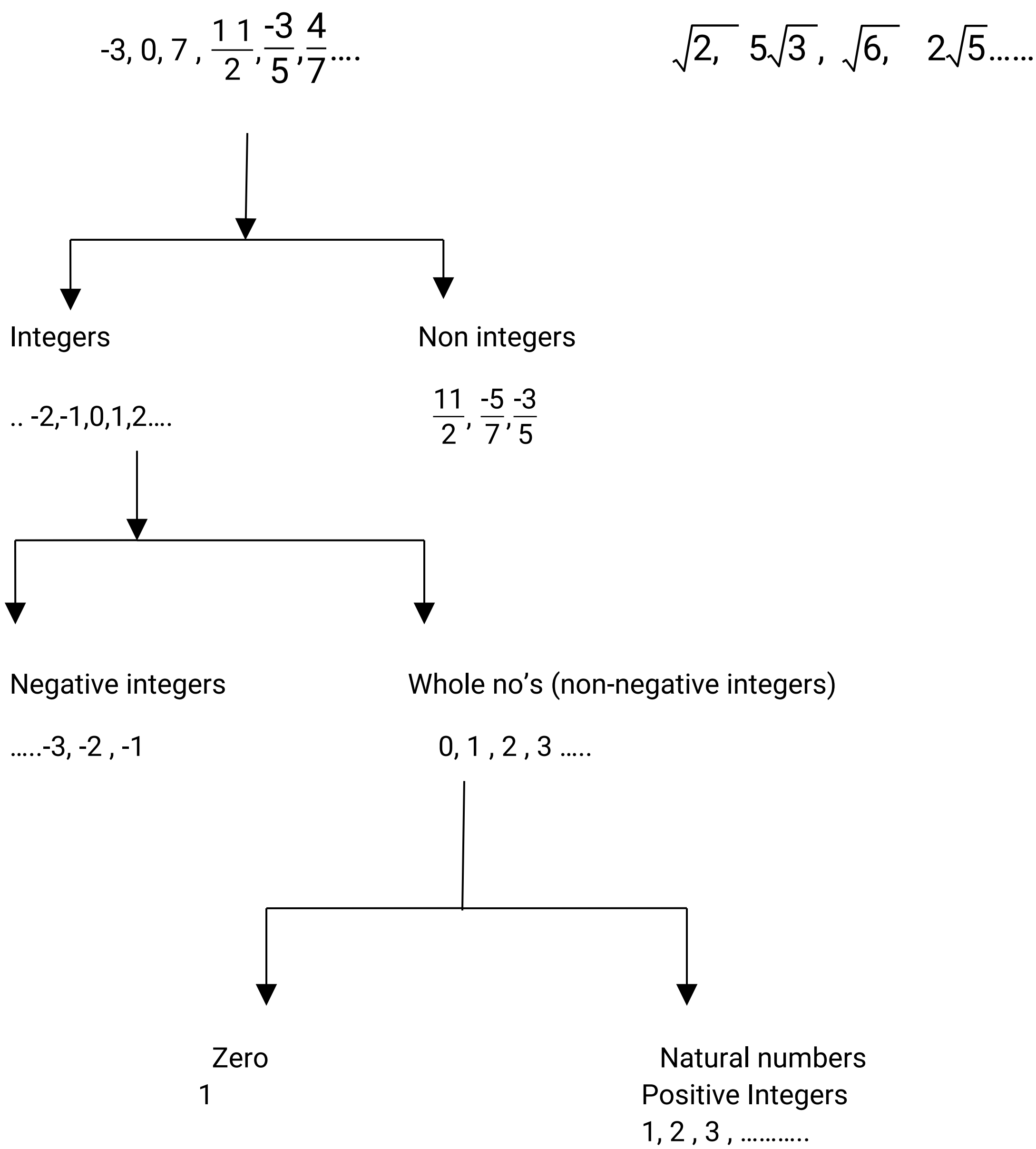
$$\text{LCM}(a, b, c) = \frac{a \times b \times c \times \text{HCF}(a, b, c)}{\text{HCF}(a, b) \times \text{HCF}(b, c) \times \text{HCF}(c, a)}$$

Revisiting Rational and Irrational Numbers , Decimal Expansion of Rational numbers.

Real numbers:

A number which is either rational or irrational is called a real number . Different components of real numbers can be understood with the help of following flow diagram





Rational numbers:- A number that can be expressed as $\frac{p}{q}$ where p, q are integers and q≠0 is called a rational number .

e.g $\frac{3}{5}$ $\frac{7}{9}$, $\frac{13}{15}$, $\frac{-7}{3}$ etc..

Irrational numbers : A number that cannot be expressed in the form $\frac{p}{q}$ where p , q are integers and q ≠0 is called irrational number.

e.g $\sqrt{2}$, $\sqrt{3}$, $\sqrt{15}$, π , $\frac{\sqrt{2}}{\sqrt{3}}$,0.1011011101111…….etc.

Co prime integers: A pair of integers having no common factors other than 1 (or -1) are called co prime integers.

e.g (1, 3) , (-2 , 5) , (6, 35) etc are co prime integers.

Theorem : Let p be a prime number and a be a positive integer .If p divides a^2 then p divides a

Given : Let p be a prime number and a be a positive integer such that p divides a^2.

To prove : P divides a.

Proof: We know that every positive integer can be expressed as the product of primes

So let a = p_1 , p_2 , …..p_n.

Where p_1, p_2, p_3 …p_n are primes not necessarily all distinct.

Then a^2 = (p_1 , p_2 … p_n) p_1 , p_2 , … p_n).

a^2 = (p_1, p_2 … p_n)

Now let p divides a^2 .

P is a prime factor of a^2.

P is one of p_1, p_2….p_n.

Since by using uniqueness of the fundamental theorem of arithmetic the only prime factor of a^2 are p_1 , p_2 … p_n

P divides a | a = p_1, p_2 … p_n|

Thus p divides a^2 | p divides a. Hence proved.

Questions : 1 Marks

Q1. Are the following statements true or false ?

(i) The number 3^n can end with the digit 0 for any $x \in N$.

Q2. (ii) Every +ve odd integer is of the form 2q +1 where q is some whole number.

Q3. Every composite number can be expressed as a product of

(A) Co primes (B) Primes (C) Twin primes (D) None .

Q4. The decimal expansion if a rational number is always

(A) Non – terminating (B) Non – terminating and non repeated (C) Terminating or non – terminating repeated (D) None

Q5. Prime factors if 4050 is

(A) $2 \times 3^3 \times 5$ (B) $2 \times 3^4 \times 5$ (C) $2 \times 3^4 \times 5^2$ (D) $2 \times 3^4 \times 5^3$

Q6. If P^2 is even integer then p is an

(A) Odd integer (B) Even integer (C) Multiple of 3 (D) None

Q7. ……………. Is a proven statement which is used to prove another statement .

(A) Lemma (B) Axiom (C) Theorem (D) Algorithm

Q8. The number $0.\overline{57}$ in the $\frac{p}{q}$ form $q \neq 0$ is

(A) $\frac{26}{45}$ (B) $\frac{13}{27}$ (c) $\frac{57}{99}$ (D) $\frac{13}{29}$

Questions : 2 Marks , 3 Marks , and 4 Marks.

1. A number when divided by 53 gives 34 as quotient and 21 as remainder .Find the number .
2. In Euclid's division lemma a= bq +r , where $0 \leq r < b$.What is a?
3. Use Euclid's division algorithm to find the HCF of (i) 650 and 1170. (ii) 870 and 225.
4. The product of two consecutive positive integers is divisible by 2. Is this statement true or false. Give reasons .
5. Use EDA to find the HCF of the following three no's 441 , 567, and 693.
6. Prove that $\sqrt{3}$ is an irrational numbers?
7. Prove that $\sqrt{2}$ is an irrational number?
8. Prove that $6+\sqrt{2}$ is irrational number.
9. Show that $(\sqrt{3}+\sqrt{5})^2$ is an irrational number?
10. What is the condition for the decimal expansion of a rational number to terminate ? Explain with an example.
11. Express the number $\overline{0.3178}$ in the form of rational number $\frac{a}{b}$.

12. Write 98 as product of its prime factors.

13. If *a* is rational and $\sqrt{b}$ is irrational, then prove that (a+ ($\sqrt{b}$) is irrational.

14. Check whether 4^n can end with the digit 0 for any natural number n.

15. Sate Unique Factorization Theorem .

16. Check whether 3^n can end with digit 0 for any natural number n.

17. In EDL , the value of r , when a +ve integer a is divided by 3 are 0 and 1 only . Is this statement true or false ? Justify your answer.

18. For what value of n, $2^n \times 5^n$ ends with 5 ?

19. The product of three consecutive positive integers is divisible by 6. Is this statement true or false? Justify your answer .

20. 21. 22. Without actually performing the long division , State whether the following rational numbers will have a terminating decimal expansion or not .Also write the terminating decimal expansion , if exist

(i) $\frac{3}{8}$ (ii) $\frac{31}{343}$ (iii) $\frac{14588}{625}$

23. If $\frac{13}{125}$ is a rational number , find the decimal expansion of it, which terminates .

24. Explain, why (3 x 5 x 7) + 7 is a composite number?

25. Express 1001 as a product if its prime factors?

26. If HCF (6, a) = 2 . LCM (6, a) = 60. Than find the value of **a**

27.State Euclid's Division Lemma .

28. Find two rational numbers and two irrational numbers between $\sqrt{2}$ and $\sqrt{3}$.

Answers of Objectives

01.True

02.True

03. Primes

04. C

05. a

06.b

07. a

08. c