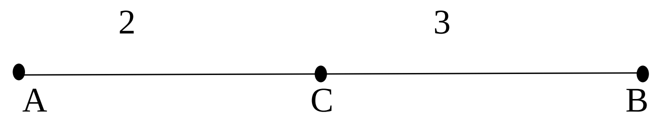
CONSTRUCTION

(Marks 4)

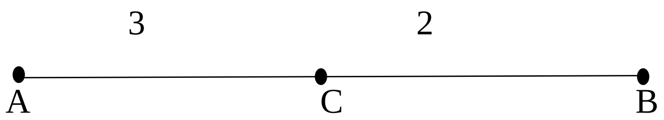
➢ *Division of a line segment and construction of a Triangle*

Division of a line Segment:

Here we will divide a given line segment AB (say) in two segments “AC” and “CB” (say), such that C divides “AB” internally in the given ratio. e.g. If AC:CB = 2:3, then C divides AB internally in the ratio 2:3.



Here, AC = $\frac{2}{3}$ AB and CB = $\frac{3}{5}$ AB.

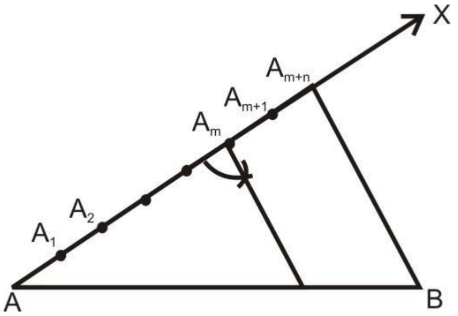


And if the ratio is 3:2, then AC = $\frac{3}{5}$ AB and CB = $\frac{2}{5}$ AB.

Construction – 1:

➢ *To divide a line segment internally in the given ratio.*

To divide a line segment AB (Say) internally in the given ratio m : n, where m and n are both positive integers. We use the following steps:



Step – 1 : Draw the given *line segment AB* and any *ray AX,* making an *acute angle* with the *line segment AB*. This *ray AX* can be *draw above* or *below AB.*

Step – 2 : Mark $m + n = p$ points ($A_1, A_2,$ ..... $A_m,$ .... A_P) on the *ray AX*, such that $AA_1 = A_1A_2 =$..... A_{P-1}, A_P.

Step – 3 : Join BA.

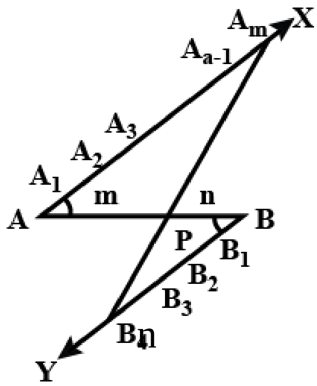
Step – 4 : Through the *point* A_m, draw a line *parallel to* A_PB (by making an *angle equal to* $<AA_PB$ *at A*) which *intersects* the *line segment AB at point C*. Thus *C divides* the *line segment AB internally* in the ratio *m : n* i.e. *AC:AB = m : n*.

Alternate Method of Construction

To divide a line segment AB (Say) internally in the given ratio m : n, where m and n are both positive integers. We use the following steps:

Step – 1 : Draw the given *line segment AB (say)* and *any ray AX* making an *acute angle* with the *line segment AB.*

Step – 2 : Draw *another ray BY // AX* by making $<ABY = < BAX$.



Step –3 : *Mark off m points* $A_1, A_2, \ldots A_n$ on *AX and n* points B_1, $B_2 \ldots$. B_n *on BY* such that $AA_1 = A_1A_2 = \ldots A_{m-1}A_m = BB_1 = B_1B_2 = \ldots B_{n-1}B_n$.

Step – 4 : Join A_mB_n which *intersects line segment AB* at the *point C.* Now, *C is required point* which divides *line segment "AB" internally* in the *ratio m : n.*

Construction – 2 :

➢ ***To construct a triangle similar to a given triangle as per the given scale factor.***

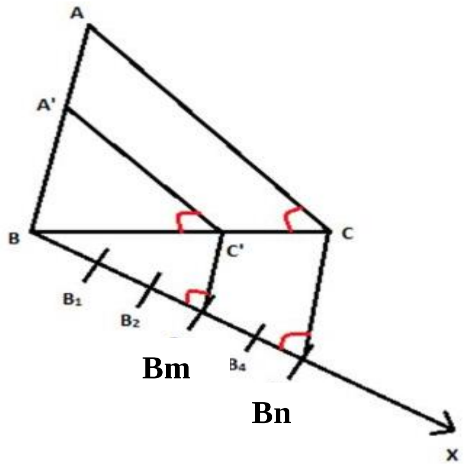
Scale Factor: *The ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle is called the scale factor*. Generally, it is written as $\frac{m}{n}$ which may be less than "1" or greater than 1.

Thus two different situations to construct a triangle depends on scale factor arise which are discussed below:

a) If $\frac{m}{n} < 1$ or "m" < n then the sides of the triangle to be constructed will be smaller than corresponding sides of the given triangle.

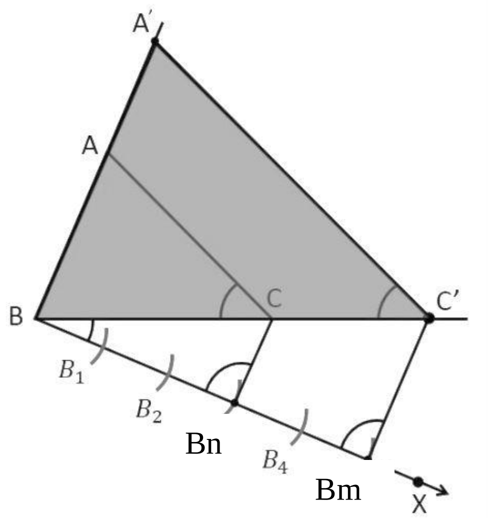
b) If $\frac{m}{n} > 1$ or m > n, then the sides of the triangle to be constructed will be larger than the corresponding sides of the given triangle.

Condition – 1 When $\frac{m}{n}$ < 1 or m < n. To construct $A^{/}BC^{/}$ ~ ABC, we use the following steps.



- ☞ Take BC as base and draw Δ ABC of given measurement.
- ☞ Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.
- ☞ Find the value of m and n, then mark off n points (the greater of m & n in $\frac{m}{n}$) on BX such that $BB_1 = B_1B_2$ …..=$B_{m1}B_n$).
- ☞ Join B_nC and draw a line through Bm (m being smaller of m & n in $\frac{m}{n}$) parallel to B_nC to intersect BC at $C^{/}$. Then $B_mC^{/}$ II "B_nC".
- ☞ Draw a line through $C^{/}$ parallel to CA which intersects BA at $A^{/}$. Then $C^{/}A^{/}$ II CA.
- ☞ Then $A^{/}$ B $C^{/}$ is the required triangle similar to given Δ ABC as per scale factor $\frac{m}{n}$ <1.

Condition – 2 When $\frac{m}{n}$ > 1 or m > n. To construct Δ $A^{/}BC^{/}$ ~ Δ ABC, when $\frac{m}{n}$ > 1, we use the following steps.



- ❖ Take BC as base and draw a Δ ABC of given measurement.
- ❖ Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.
- ❖ Find the value of m and n, then mark off m points (the greater of m & n in $\frac{m}{n}$) B_1B_2….. B_m on BX such that BB_1=B_1 B_2 …..=B_{m-1} B_m).
- ❖ Join B_nC (n being smaller of m & n in $\frac{m}{n}$) and draw a line through B_m II BC to intersect the extended line segment BC at $C^{/}$.
- ❖ Draw a line through $C^{/}$ parallel to CA intersecting the extended line segment BA at $A^{/}$.
- ❖ Then $A^{/}$ B $C^{/}$ is the required triangle.

IMPORTANT CONSTRUCTIONS OF A TRIANGLE IN DIFFERENT CASES

Before constructing a triangle similar to a given triangle say Δ ABC, we have to construct the Δ ABC. For this, let us review some constructions in different cases which are given below:

CASE – 1 : When three sides are given.

We use the following steps of construction:

Step – 1 : First draw base BC.

Step – 2 : By taking B and C as centers, draw two arcs of radius CA and BA respectively which intersects each other at A.

Step – 3 **:** Join AB and AC.

Then ABC is a required triangle.

CASE – 2 : When base and alternate of a triangle are given.

Let the base of Δ ABC be BC = m cm and Alternate is n cm. Then, we use the following steps.

Step – 1 : Draw the base BC = m cm and draw its perpendicular bisector OQ (say) which intersects BC at P (say).

Step – 2 : By taking P as center, draw an arc of radius n cm which intersects the line segment PO or extended line segment of PO at A.

Step – 3 : Join AB and AC.

Then ABC is a required triangle.

CASE – 3 : When two sides and angle between them are given.

Let the two sides of Δ ABC be AB = a cm, BC = b cm and angle between them i.e. <ABC = x^0. Then, we use the following steps.

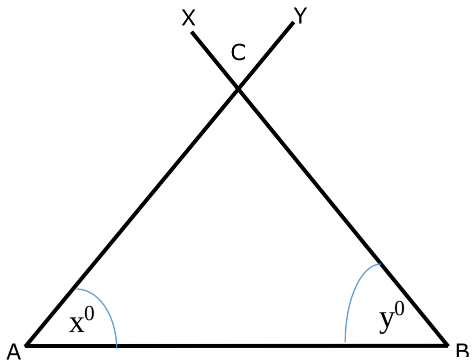
Step – 1 : Draw base "AB" = "a" cm

Step – 1 : Draw a ray "BX" making an angle x^0 at B and cut off BC = b cm from BX.

Step – 3 : Join "AC". Thus Δ ABC is required triangle.

CASE – 3 : When one side and two angles are given.

Let one side of triangle ABC be AB = a cm and $< A = x^0$ and $< B = y^0$



Step – 1 : Draw base AB = a cm

Step – 2 : Draw two rays AX and BY making an angle x^0 at "A" and y^0 at B which intersects each other at C.

Step – 3 : Join AC and CB.

Then ABC is a required triangle.

CONSTRUCTION OF TANGENTS TO A CIRCLE

Tangent: ***A tangent to a circle is a straight line which touches the circle at a point. This point is called point of contact and the radius through the point of contact is perpendicular to the tangent.*** The number of tangents drawn to a circle from a point depends on the position of the point with respect to circle.

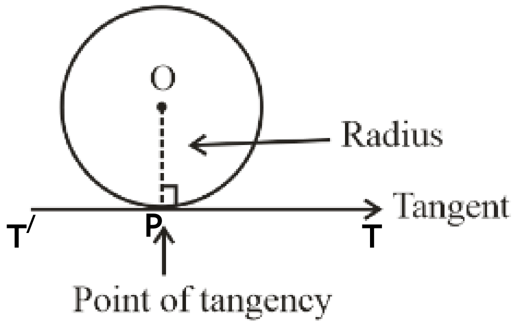
a) If a point lies on the circle, then only one tangent at this point can be drawn.

b) If a point lies inside the circle, then no tangent can be drawn.

c) If a point lies outside the circle, then two tangents from this point can be drawn.

Construction of a tangent to a circle at a point lies on it. (By two methods)

Method – 1 : By using the Centre of Circle.



Step – 1 : Take a point O as center and draw a circle of given radius.

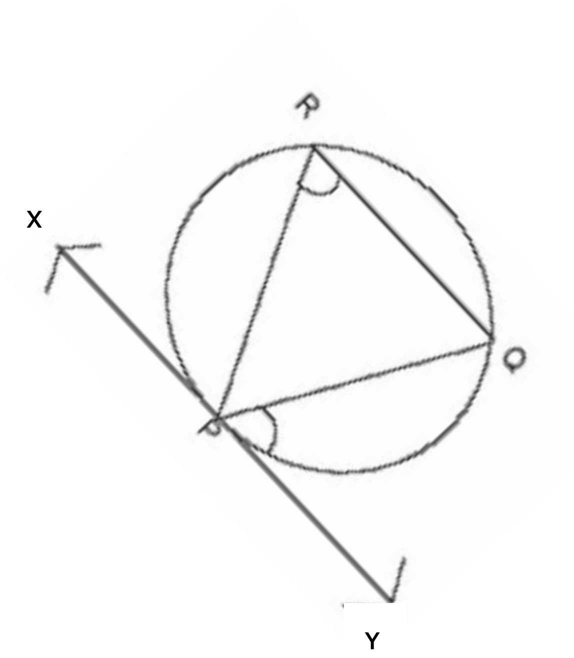
Step – 2 : Take a point P on the circle, at which we want to draw a tangent.

Step – 3 : Join OP, which is the radius of Circle.

Step – 4 : Take OP as base and construct $< OPT = 90^0$ at P.

Step – 5 : Produce "TP to T/ to get the required tangent TPT/

Method – 1I : ***Without using the center of Circle.***



Step – 1 : Draw a circle of given radius r cm and take a point P (at which we want to draw a tangent) on the circle.

Step – 2 : Draw any chord PQ through the given point P on the circle.

Step – 3 : Take a point R in either the major arc or minor arc and join PR and QR.

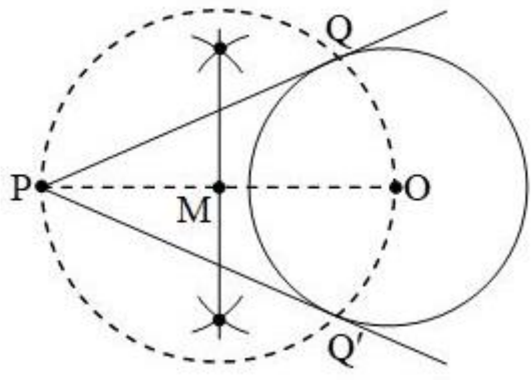
Step – 4 : Taking PQ as base, construct $<$ QPY equal to $<$ PRQ and on the opposite side of "R".

Step – 5 : Produce YP to X to get the tangent XPY.

Construction of Tangents to a circle from a point outside the circle:

If a point lies outside the circle, then there will be two tangents to the circle from this point. These tangents can be drawn in two cases which are given below:

CASE – 1 : When the center of circle is given.



Step – 1 : Draw a circle with center O and take a point P outside it.

Step – 2 : Join OP and bisect it. Let its mid-point be M, then MP = MO.

Step – 3 : Taking M as center and MO or MP as radius draw a dotted circle which intersects the given circle at Q and $Q^{/}$ (say).

Step – 4 : Join PQ and $PQ^{/}$.

Thus PQ and $PQ^{/}$ are the required tangents drawn to the circle from the external point P. Here we observe that $PQ = PQ^{/}$.

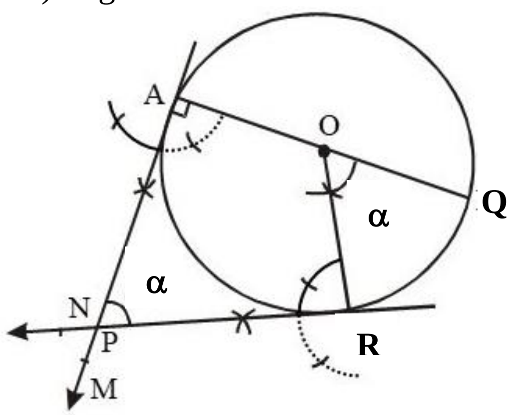
Case – II : When the center of the circle is unknown.

Step – 1 : First draw the circle and then two non-parallel chords of the circle.

Step – II : Draw the perpendicular bisectors of both chords which intersect each other at a point say O. Then this point O gives the center of given circle. Now use the steps given in case – 1 to draw the tangents

Construction of tangents to a circle when angle between them is given

Sometimes, angle between two tangents (or pair of tangents) is given and we have to draw the tangents. Then, we use following steps of constructions:



Step – 1 : First, draw the given circle with center O and radius r cm.

Step – 2 : Draw any diameter say AOQ of this circle.

Step – 3 : Make given angle α at center O with OQ (say) as base which intersects the circle at point R (say) or draw the radius or meet the circle at R such that $< QOR = \alpha$

Step – 1 : Now, draw perpendiculars to OA at A" and OR at R which intersects each other at a point say P.

The AP and RP are the required pair to tangents to given circle inclined at angle α.

EXERCISE

Q.1 Divide a line segment of length 9.6 cm in the ratio of 5:3. Measure the two parts.

Q.2 Draw a line segment of length 7.7 cm and divide it in the ratio 3:4.

Q.3 Construct a triangle similar to given triangle ABC, where AB = 6 cm, BC = 7 cm and AC = 8 cm with its sides equal to $\frac{3}{4}$ of the corresponding sides of triangle ABC.

Q.4 Construct a triangle similar to a given triangle ABC whose sides are 6 cm, 7 cm and 8 cm with its sides equal to $\frac{5}{3}$ of the corresponding sides of a triangle ABC.

Q.5 Draw an equilateral triangle ABC of each side 4 cm. Construct a triangle similar to it and of scale factor $\frac{3}{5}$. Is the new triangle also an equilateral? *(Yes No)*

Q.6 Draw two tangents to a circle of radius 4 cm from a point "P" at a distance of 7 cm from its center.

Q.7 Draw two tangents from the end points of the diameter of a circle of radius 4 cm. Are these tangents parallel? *(Yes / No)*

Q.8 Draw a circle of radius 1 cm. From a point P, 2.2 cm apart from the center of the circle.

Q.9 Construct a pair of tangents to circle whose radius is 6.5 cm are inclined to each other at angle of 30^0.

Q.10 Draw a pair of tangents to a circle of radius 3 cm which are inclined to each other at an angle of 45^0.

Q.11 Construct a Δ ABC in which BC = 13 cm, CA = 5 cm, AB = 12 cm. Draw its incircle and measure it radius.

Q.12 Draw a pair of tangents to circle of radius 3 cm that are inclined to each other at an angle of 50^0.

Q.13 **Value Based Questions**

a) Sanjeev have a piece of cloth of 8 cm long. He decided to divide this piece into two "A" and "B" internally in the ratio of 3:4.

i) Draw a construction of above problem.

ii) If Sanjeev gave 4^{th} part of the piece of cloth to the person "A", then what value is violated by Sanjeev?

Q14 Geometrically divide line segment of length 8.4 cm in the ratio of 5:2.

Q.15 Construct a Δ ABC similar to a given isosceles Δ PQR with QR = 6 cm, PR=PR=5 cm such that each side is $6/7^{th}$ of the corresponding sides of Δ PQR.

Q.16 Draw a circle of diameter 8 cm, from a point "P" 7 cm away from its center. Construct a pair of tangents to the circle.

Q.17 Draw a right angled triangle ABC, in which BC = 12 cm, AB = 5 cm $<$ B= 90^0. Then construct a triangle similar to it and of scale factor 2/3, is the new triangle also a right angle triangle.

Q.18 Draw a circle of radius 4 cm. Construct a pair of tangents on it, so that they are inclined at 60^0.

Q.19 Draw a circle of radius 4 cm. Take two points P & Q on one of its extended diameters, each at a distance of 9 cm from its center. Draw tangents to the circle from these two points "P & Q".

Q.20 Construct a triangle ABC, in which AB = 5 cm, <A = 300 and AC = 6 cm. Then construct a triangle whose sides are 4/3 of the corresponding sides of triangle ABC.

Q.21 Draw a triangle ABC with BC = 7 cm , $<$ B = 450 & $<$ C = 600. Then construct another triangle whose sides are 3/5 times of the corresponding sides of triangle ABC& justify your construction.

1 Mark Questions

Q1. A line segment of length 20 cm is divided in the ratio of 2:3, the measure of the two parts in the given ratio respectively would be

a) 8 cm, 12 cm b) 4 cm, 8 cm c) 4 cm, 12 cm d) 4cm, 9 cm

Q.2 A line segment of length 25 cm in divided in the ratio2:3, the measure of the two parts in the given ratio respectively would be

a) 15 cm, 10 cm b) 10 cm, 15 cm c) 9 cm, 6 cm d)None

Q.3 To construct a pair of tangents to a circle at an angle of 600 to each other, it is to draw tangents at end points of those two radii of the circle, the angle between them should be

a) 100^0 b) 90^0 c) 180^0 d) 120^0

Q.4 A pair of tangents can be constructed from a point to a circle of radius 3.5 cm situated at a distance of ………….. from the center

a) 3.5 cm b) 2.5 cm d) 5 cm d) 2 cm

Q.5 To construct a triangle ABC and then a triangle similar to it whose sides are 2/3 of the corresponding sides of the first triangle. A ray "AX is drawn where multiple points at equidistance are located. The last point to which point "B" will meet the ray AX will be

a) A_1 b) A_2 c) A_3 d) A_4

Q.6 To draw a pair of tangents to a circle which are inclined to each other at an angle of 45^0. It is required to draw tangents at the end points of those two radii of the circle, the angle between

a) 135^0 b) 155^0 c) 160^0 d) 120^0

Q.7 How many tangents can be drawn from a point out to a circle?

a) 1 b) 2 c) 3 d) None

Q.8 The lengths of two tangents drawn from a point outside to a circle are ……….. (equal /not equal).

ANSWER

Q.1	(a)	Q.2	(b)	Q.3	(d)	Q.4	(c)
Q.5	(c)	Q.6	(a)	Q.7	(b)	Q.8	equal