PROBABILITY

Introduction:

In life we usually come across two types of experiments namely "Deterministic Experiment and Random Experiment".

Deterministic Experiment:

An experiment is said to ""Deterministic", it has unique outcome, when repeated under same conditions e.g.

- 1. Na + Cl → Nacl
- 2. $2H_2 + O_2 \longrightarrow 2H_2O$
- 4. Every soul shall taste death.
- 5. Time and Tide wait for none.

Such experiments are full of certainty.

Radom Experiment

An experiment which has two or more than two outcomes is said to be "Random" or "Probabilistic Experiment". The outcomes of such experiment face uncertainty.

- 1. On a cloudy day our mothers provide us an umbrella, when we leave our homes, because it seems that it may rain.
- 2. Weather forecasting is another applicative example of Random experiment.
- 3. Tossing a coin.
- 4. Match between two teams.
- 5. Rolling a dice.

Probability is the measurement of certainty or uncertainty of outcomes in a Random experiment, where there is uncertainty, Probability steps in:

Sample Space: The set of all possible outcomes of a Random experiment is called sample space. It is usually denoted by "S".

1. Rolling a die:

$$S = \{1, 2, 3, 4, 5, 6\}$$

2. Rolling a pair of die:

$$S = \{(1,1),(1,2), (1,3), (1,4), (1,5), (1,6)\}$$

$$\{(2,1),(2,2), (2,3), (2,4), (2,5), (2,6)\}$$

$$\{(3,1),(3,2), (3,3), (3,4), (3,5), (3,6)\}$$

$$\{(4,1),(4,2), (4,3), (4,4), (4,5), (4,6)\}$$

$$\{(5,1),(5,2), (5,3), (5,4), (5,5), (5,6)\}$$

$$\{(6,1),\{6,2), (6,3), (6,4), (6,5), (6,6)\}$$

Tossing a Coin:

$$S = \{HT\}$$

4 Tossing two coins

$$S = \{(HH), (HT), (TH), (TT)\}$$

5 Tossing three coins

6 Tossing four coins

$$S = \{HHHH, HHHT, HHTH, HTHH\}$$
 $\{HHTT, HTTH, HTHT, HTTT\}$
 $\{TTTT, TTTH, TTHT, THTT\}$
 $\{TTHH, THHT, THTH THHH\}$

(Write these elements, then replace "H" by "T" and vice versa)

Remarks:

1. In case of rolling "n" dies

Total elements $= 6^{n}$

2. In case of Tossing "n" coins

Total elements $= 2^n$

7 Tossing a coin and die

$$S = \{ H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6 \}$$

EVENT: Let "S" be a sample space associated with the Random experiment then any part (subset) of "S" is called "*Event*". e.g. In case of rolling a dice

$$S = \{1, 2, 3, 4, 5, 6\}$$

Let "A" be the *event* getting *even number*. Then

$$A = \{2, 4, 6\}$$

Let "B" be the event getting Prime number. Then

$$B = \{2, 3, 5\}$$

(Compound even containing more than one element)

Let "C" be the event getting even number. Then

$$C = \{2\}$$

(Elementary event /simple event containing one element)

Let "D" be the event getting number less than "7". Then

$$D = \{1, 2, 3, 4, 5, 6\}$$

(Certain / sure elements, containing all elements of sample space).

Impossible Event:

Let "E" be an event getting number less than "1", which is impossible, this is called Impossible or Null Event denoted by \emptyset containing no element at all.

Complimentary Event (\overline{A}) :

Let "S" be a sample space, in case of match between two teams "E" and "F", if "A" is the event winning match by team "E" then complement event of "A" denoted by \overline{A} means losing match by team "E". e.g.

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{2, 4, 6\}$$

Then \overline{A} is also called "Negation of Event "A" or not "A". Thus

$$\overline{A} = \{1, 3, 5\}$$

Remarks: \overline{A} is also called "negation of event" "A" or not "A".

Occurrence of an Event:

An event "A" associated to a "Random experiment" is said to occur if any one of the elementary events associated to the even "A" is an outcome. e.g.

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{2, 4, 6\}$$

Suppose that in Trial/Experiment, the outcome is "4". We say event "A" has occurred.

In another trial, if outcome is "3", then we say even "A" has not occurred.

CLASS – X

CHAPTER - 15

IMPORTANT FORMULAS & CONCEPTS

PROBABILITY:

Experimental or empirical probability "P(E)" of an even "E" is

$$P(E) = \frac{Number of trials in which the event happened}{Total number of Trials}$$

The theoretical probability (also called classical probability of an event "A", written as "P(A)" is

$$\mathbf{P}(\mathbf{A}) = \frac{Number\ of\ outcomes\ of\ favourable\ to\ A}{Number\ of\ all\ possible\ outcomes\ of\ the\ experiment}$$

$$= \frac{n\ (A)}{n\ (S)}$$

Two or more events of an experiment, where occurrence of an event prevents of all other events are called "Mutually Exclusive Events."

Compliment Events and Probability:

We denote the event "not E" by \overline{E} , so

$$P(E) + P(not E) = 1.$$

i.e.
$$P(E) + P\overline{E} = 1$$
, which gives us $P(\overline{E}) = 1 - P(E)$

In general, it is true that for an even E

$$P(\overline{E})$$
 = 1 - $P(E)$

The probability of an event which is impossible to occur is 0. Such event is called *impossible event, dentoted by P*(\emptyset). Thus

$$P(\emptyset) = \frac{0}{n(s)} = 0$$
 Least (minimum)

The probabilty of an event which is sure (or certain) to occur is "1". Such an event is called *Sure Event* or a *Certain Event*, denoted by P(S). Thus

$$P(S)$$
 = $\frac{n(S)}{n(S)}$ = 1 Maximum

- The probability of an event "E" is a number P(E) such that $0 \le P(E) \le 1$
- An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is 1.

Deck of Cards and Probability

A deck of playing cards consists of 52 cards which are divided into 4 suits of 13 cards each. They are Black Spades () Red Hearts (), Red Diamond () and Black Club ().

The cards in each suit are Ace, King, Queen, Jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2.

Kings, Queen and Jacks are called Face Cards.

Example set of 52 poker playing cards

	Suit	Ace	2	3	4	5	6	7	8	9	10	Jack	Queen	King
*	Clubs	*	2 *	* *	:+ + + +;	** * * * *;	\$4 4 4 4 4 4;	7++ ++ ++2	***	**** *** * *;	***** ******	- E	2	K S
♦	Diamonds	•	2	* *	?• • • •;	\$. .	5 * * * * * * * *	7 · · · · · · · · · · · · · · · · · · ·		***************************************	io de la constantina della con	. B.	2 6	, i
Y	Hearts	•	₹ ∀	₹ ₩	# ¥ ¥	ξΨ Ψ Ψ Δ Δ;	\$₩ ₩ ₩ ₩ A A ;	2 4 4 2	**************************************	**************************************	10 W W	; 5	2.	ŧ.
•	Spades	² ♠ .	* •	÷	:4	;◆	% ↔ ♦ ♦ ♦ ♦ ;	! ♦ ♦ • • •:	;	****	****		2 2	· D

Suits	Spade 🆍	Club 👫	Diamond (Heart 💙			
Cards	13	13	13	13			
Colour	Black (2	6 Cards)	Red (26 Cards)				
Face Card (12)	J, K, Q	J,K, Q	J,K,Q	A, J, K, Q (Total face Cards= 12)			
Court Cards	A,J, K, Q	A, J,K, Q	A, J,K,Q	A, J, K, Q (Total face Cards= 16)			

Equally Likely Events: Two or more events are said to be equally likely, if each one of them has an equal chance of occurrence.

Mutually Exclusive Events: Two or more events are mutually exclusive, if the occurrence of each event prevents the every other event.

Complementary Events: Consider an event has few outcomes. Event of all other outcomes in the sample survey which are not in the favourable even is called complementary event.

Exhausive Events : All the events are exhausive events, if their union is the sample space.

Sure Events: The sample space of a random experiment is called *sure or certain event* as any one of its elements will surely occur in any trail of the experiment.

Impossible Event : An event which will occur on any account is called an *impossible event*.

1 Mark Questions

Q.1					560 times ead" in this		d" occurs. The empirio				
	a)	0.5	b)	0.56	c)	0.44		d)	0.056		
Q.2	Two coins are tossed 200 times and the following out comes are recorded										
		F	IH		HT/TH			TT			
		4	56		110		34				
	Wha	it is the e	empirical pr	obability o	f occurrence	e of at le	east one "head" in the case				
	a)	0.33	b)	0.34	c)	0.66		d)	0.83		
	A die is thrown 200 times and the following outcome are noted, with the frequencies										
	Outcome		1	2	3 4		5		6		
	Fre	quency	56	22	30	42		32	18		
Q.3	Wha	at is the ϵ	empirical pr	obability o	of getting a 1	I in the	abo	ve case.			
	a)	0.28	b)	0.22	c)	0.15		d)	0.21		
Q.4	Wha	it is the e	empirical pr	obability o	of getting a n	number	less	than 4?			
	a)	0.50	b)	0.54	c)	0.46		d)	0.52		
Q.5	Wha	it is the e	empirical pr	obability o	of getting a	number	grea	ater than 4.			
	a)	0.32	b)	0.25	c)	0.18		d)	0.30		
Q.6	On a	n particul	lar day, the	number of	vehicles pa	ssing a	cros	ssing is give	en below:		
	Vel	hicle	Two wh	eeler	Three who	eeler	Four wheeler				
	Frequency 52 71 77										
	What is the probability of a two wheeler passing the crossing on that day?										

c)

0.385

0.615

d)

0.71

b)

0.26

a)

Q.7 The following table shows the blood- group of 100 students

Blood group	A ^{+ve}	B ^{-ve}	O ^{+ve}	AB-ve	B ^{+ve}
Number of students	12	23	35	20	10

One student is taken a random. What is probability that his blood group is B^{+ve}

- a) 0.12
- b) 0.35
- c) 0.20
- d) 0.10

Q.8 In a bag, there are 100 bulbs, out of which 30 are bad ones. A bulb is taken out of the bag at random. The proability of the selected bulb to be good is

- a) 0.50
- b) 0.70
- c) 0.30
- d) None of these

Q.9 On a page of telephone directory having 250 telephone numbers, the frequency of the unit digit of those number is given below

0	1	2	3	4	5	6	7	8	9
18	22	32	28	40	30	30	22	18	10

A telephone number is selected from the page at random. What is the probability that its digit is

(a)2

- a) 0.16
- b) 0.128
- c) 0.064
- d) 0.04

(b) More than 6

- a) 0.20
- b) 0.25
- c) 0.32
- d) 0.16

(c) less than 2

- a) 0.16
- b) 0.18
- c) 0.22
- d) 0.32

Q.10 10 defective pens are accidentally mixed with 90 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is good one.

- a) 0.10
- b) 0.20
- c) 0.90
- d) 1.0

- Q.11 Define Probability
- Q.12 Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a football game.
- Q.13 Probability of sure event is 1 (T/F)

Q.5 (b) Q.1 (b) Q.2 (d) Q.3 (a) **Q**.4 (b) Q.6 (a) Q.7 Q.9(a) b Q.9 (c) a Q.9 (c) a (d) **Q**.8 (b) Q.10 (c) Q.11 Definition Q.12 Because H and T have equal probabilities to occur Q.13 (T)

2 Mark Questions

(ii)

a multiple of 3

An unbaised die is thrown. What is the probability of getting

Q.1

i)

An even number

	iii) v) vii)	An even number A number betwee A number less t	een 2 and	6	(vi)			lber less than 1.			
Q.2	Two unbaised coins are tossed simultaneously. Find the probability of getting										
	i) iv)	Two heads Atleast one head	ii) d v)			,	One '				
Q.3	Three unbaised coins are tossed together. Find the probability of getting										
	i) iv)	All heads Atleast two head	ii) ds v)			iii)	One ?	Head			
Q.4	Two dice are thrown simultaneously. Find the probability of getting										
	i) iii) iv)	An even numbe A total of at least A doublet and (Hint $A = \{ (1,1) \}$ P(A)									
	v) vi)	A multiple of 2 A multiple of 3		e and a multip	le of 3	on the	other.				
Q.5		e are six marbles ing a marble with			to 6. V	Vhat is	s the pr	obability of			
Q.6	•	orobability that it vain tommorrow?	will rain to	mmorrow is 0	.75. WI	nat is th	ne prob	ability it will			
Q.7		rn contain 10 readability that the bal			ball is	drawn	at rand	om. Find the			
Q.8		g contains 3 red bom from the bag.	*					l is drawn at			
	i)	White ii)		iii) ge 10 of 17	Black	ζ.	iv)	Not Red			

- If probabity of winning a game is 0.5. What is the probability of lossing it. Q.9
- Q.10 In a lottery there are 10 prizes and 25 blanks. What is he probability of
 - getting a prize i)
- not getting a prize ii)
- Q.11 What is the probability that a number selected from the numbers 1,2,3.... 20 is a mulitiple of 3.

Q.1 i)
$$\frac{1}{2}$$
 ii) $\frac{1}{3}$ iii) $\frac{2}{3}$ iv) $\frac{1}{2}$ v) $\frac{1}{2}$ vi) 0 vii) 1

Q.2 i)
$$\frac{1}{4}$$
 ii) $\frac{1}{2}$ iii) $\frac{1}{2}$ iv) $\frac{3}{4}$ v) $\frac{3}{4}$ vi) $\frac{1}{4}$

Q.3 i)
$$\frac{1}{8}$$
 ii) $\frac{3}{8}$ iii) $\frac{3}{8}$ iv) $\frac{1}{2}$ v) $\frac{7}{8}$

Q.4 i)
$$\frac{1}{2}$$
 ii) $\frac{5}{12}$ iii) $\frac{1}{6}$ iv) $\frac{1}{6}$ v) $\frac{1}{12}$ vi) $\frac{11}{36}$ Q.5 $\frac{1}{2}$ Q.6 0.25 Q.7 $\frac{4}{9}$ Q.8 i) $\frac{1}{3}$ ii) $\frac{1}{4}$ iii) $\frac{5}{12}$ iv) $\frac{3}{4}$

Q.5
$$\frac{1}{2}$$
 Q.6 0.25 Q.7 $\frac{4}{9}$ Q.8 i) $\frac{1}{3}$ ii) $\frac{1}{4}$ iii) $\frac{5}{4}$

Q.9 0.95 Q.10 i)
$$\frac{2}{7}$$
 ii) $\frac{5}{7}$ Q.11 $\frac{3}{10}$

3 MARK QUESTIONS

- Q.1 A lot consits of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy it if it defective. The shopkeeper draws one pen at random and gives it to her. What is a probability that
 - a) She will buy it
- b) She will not buy it
- Q.2 A jar contains 54 marbles each of which some are blue, some are green and some are white. The probability of selecting a green marble at random is 1/3 and the probability of selecting a blue marble at random is 4/9. How many white marbles does the jar contain. (*Hint:* $P(G) + P(B) + P(W) = \{1\}$
- Q.3 A letter is chosen at random from the letter of the word "ASSASSINATION". Find the probability that letter choosen is (i) a vowel (ii) an consonant (iii) A (iv) S (v) N.
- Q.4 A letter is choosen at random from the letter of the word "INDEPENDENCE". Find the probability that the letter choosen is a (i) Vowel (ii) Consonant (iii) I (iv) N (v) D.
- Q.5 A letter is choosen at random from the letter of the word "MATHEMATICS". Find the probability that the letter choosen is a (i) Vowel (ii) Consonant (iii) A (iv) T (v) M.
- Q.6 A letter of English alphabet is choosen at random. Detrmine the probability that the letter is consonant.
- Q.7 There are 1000 sealed envelopes in a box . 10 of them contain a cash prize of Rs. 100. 100 of them contain a cash prize of Rs. 50 each and 200 them contain a cash prize of Rs. 10 and rest do not contain any cash prize. If they are well shuffled and an envelope is picked up out. What is the probability that it contains no cash prize?
- Q.8 Box "A" contains 25 slips of which 19 are marked Rs. 1 and other marked Rs. 5 each. Box "B" contains 50 slips of which 45 are marked Rs. 1, each and other are marked Rs. 13 each. Slips of both boxes are poured into a third box and reshuffled. A slip is drawn at random. What is the probability that it is marked other than Rs. 1?

- Q.9 A carton of 24 bulbs contain 6 defective bulbs. One bulb is drawn at random. What is the probability that the bulb is not defective. If the bulb selected is defective and it is not replaced and a second bulb is selected at random from the rest. What is the probability that the second is defective?
- Q.10 A child's game has 8 triangles of which three are blue and rest are red and 10 squares of which six are blue and rest are red. One piece is lost at random. Find the probability it is a
 - (i) triangle (ii) square (iii) square of blue colur (iv) triangle of red colour
- Q.11 In a game the entry fee is Rs. 5. The game consists of a tossing a coin three times. If one or two head show. Sweta gets her entry fee back. If she throws three heads, she receives double the entry fees. Otherwise, she will loss. For tossing a coin three times. Find the probability that she
 - (i) loses the entry fee (ii) gets double entry fee
 - (iii) Just gets her entry fee.
- Q.12 A dice has its six faces marked 0, 1, 1, 1, 6, 6. Two such dice are thrown together and the total score is recored
 - (i) how many different scores are possible?
 - (ii) What is the probability of getting a total seven?

(Hint: Different Scores: 0,1,2,6,7,12.

Here "S" =
$$(0,0)$$
 $(0,1)$, $(0,1)$ $(0,1)$, $(0,6)$, $(0,6)$, $(1,0)$, $(1,1)$, $($

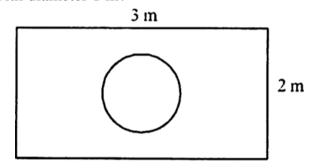
$$P(Total) = 12/36 = 1/3$$

- Q.13 A lot consists of 48 mobile phones of which 42 are good, three have only minor defect and three have major defects. Varnika will buy a phone, if it is good, but the trader will only buy a mobile if it had no major defect. One phone is selected at random from the lot. What is the probability that it is
 - (i) a good phone (ii) a bad phone
- Q.14 i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective.

- ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective.
- Q.15 A game of chance consits of spinnings an arrow which comes to rest pointing at one of the numbers, 1, 2, 3, 4, 5, 6, 7, 8 (see fig) and these are equally likely outcomes. What is the probability that it will point at (i) 8? (ii) an odd number? (iii) a number greater than 2? (iv) a number less than 9?.



Q.16 Suppose you drop a die at random on the rectangular region shown in above right sided figure. What is the probability that it will land inside the circle with diameter 1 m?



.17 A child has a die whose six faces show the letters as given below:



The die is thrown once. What is the probability of getting (i) A? ii) D?.

Q.18 A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanief wins if all the tosses give the same result i.e. three heads or three tails, and loses otherwise. Calculate the probability that Hanief will lose the game.

Q.1 (i)
$$\frac{31}{36}$$
 (ii) $\frac{5}{36}$ Q.2 (i) $\frac{2}{9}$

Q.3 (i)
$$\frac{6}{13}$$
 (ii) $\frac{7}{13}$ (iii) $\frac{3}{13}$ (iv) $\frac{4}{13}$ (v) $\frac{2}{13}$

Q.4 (i)
$$\frac{5}{12}$$
 (ii) $\frac{7}{12}$ (iii) $\frac{1}{3}$ (iv) $\frac{1}{4}$ (v) $\frac{1}{6}$

Q.5
$$\frac{4}{11}$$
 (ii) $\frac{7}{11}$ (iii) $\frac{2}{11}$ (iv) $\frac{2}{11}$ (iii) $\frac{2}{11}$

Q.6
$$\frac{21}{26}$$
 Q.7 $\frac{69}{100} = 0.69$ Q.8 $\frac{11}{75}$ Q.9 (i) $\frac{3}{4}$ (ii) $\frac{5}{23}$

Q.10 (i)
$$\frac{4}{9}$$
 (ii) $\frac{5}{9}$ (iii) $\frac{1}{3}$ (iv) $\frac{5}{18}$

Q.11 (i)
$$\frac{1}{8}$$
 (ii) $\frac{1}{8}$ (iii) $\frac{3}{4}$ Q.12 (i) six (ii) $\frac{1}{3}$

Q.13 (i)
$$\frac{14}{15}$$
 (ii) $\frac{1}{15}$ Q.14 (i) $\frac{1}{5}$ (ii) $\frac{15}{19}$

Q.15 (i)
$$\frac{1}{8}$$
 (ii) $\frac{1}{2}$ (iii) $\frac{3}{4}$ Q.16 (i) $\frac{\pi}{24} = \frac{11}{84}$

Q.17 (i)
$$\frac{1}{3}$$
 (ii) $\frac{1}{6}$ Q.18 $\frac{3}{4}$

4 Mark Questions

Q.1	Find the probability that a leap year selected at Random will contain 53 Sundays (Hint" In a leap year, we have 366 days or 52 weeks and 2 days. The remaining 2 days can be										
	a) c)	Sundays and Tuesday and	Wedn	esday			Monday and Tuesday Wednesday and Thursday				
	e) g)	Thursday and Saturday and		•	•			Friday and Saturday			
	Hence required probability $=\frac{2}{7}$										
Q.2		card is drawn drawn. Find t		•				he 52 c	ards be	eing equ	ıally
	a)	An ace		b)	Red		c)			r King	
	d) g)	Red and a Kanana 2 of Diamon	-	e) h)		e card Black S	f) Suit.	A Red	d face o	card	
Q.3	and th	King,Queen an nen well shuff bility of:							_	•	
	a)	a Heart	b)	a King	g	c)	a Clul	o d)	the 10	of hear	rts
Q.4	What	is the probabi	lity tha	t oridn	ary yea	r has 5	3 Sund	ays?			
Q.5		Queens and Bl s drawn at Ran n is					_			_	
	a)	a King	b)	of Red	d colou	r c)	a Face	e card	d)	a Que	en
Q.6		drawn in a b this bag. Find	•						s drawı	n at ran	dom
	a) c)	not divisible not a perfect	•	numbe	er	b)	a prin	ne num	ber gre	eater tha	an 7
Q.7	A dic	e is rolled twic	ce. Find	d the pr	obabili	ty that					
	a)	5 will not come up either time					5 will come up exactly one time				

- Q.8 A box contains cards numbered 3, 5, 7, 9,, 35, 37. A card is drawn at random from the box. Find the probability that the number on the drawn is a prime number.
- Q.9 What is the probability that a number selected at random from the numbers 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, will be their average.
- Q.10 If a number x is choosen at random from the numbers -2, -1, 0, 1, 2. What is the probability that $x^2 < 27$.
- Q.11 A box contains 100 red cards, 200 yellow cards and 50 blue cards. If a card is drawn at random from the box. Find the probability that it will be
 - a) a blue card b) not a yellow card c) neither yellow nor blue card

	ANSWERS												
<i>(</i> :	Q.1	<u>2</u> 7	Q.2	i)	1 13	ii)	1 2	iii)	7 13	iv)	1 26	v)	3 13
	vi)	3 26	vii)	1 52	viii)	1 26	Q.3	i)	13 49	ii)	3 49	iii)	10 49
	iv)	<u>1</u> 49	Q.4	i)	<u>1</u> 7	Q.5	i)	1 12	ii)	<u>1</u> 2	iii)	$\frac{1}{6}$ iv	$(1)\frac{1}{24}$
	Q.6	i)	<u>2</u> 3	ii)	<u>1</u> 5	iii)	<u>5</u> 6	Q.7	i)	25 36	ii)	5 18	
	Q.8	i)	<u>5</u> 9	Q.9	i)	3 10	Q.10	i)	<u>3</u> 5	Q11	i)	1 7	
	ii)	3 7	iii)	2 7									
	V. 1. 1. 1. 1.	7.7.7.7.7.7.											