Chapter 2

POLYNOMIALS

INTRODUCTION:

Expressions like 4x+2, $2y^2-3y+4$, $5x^3-4x^2+x-\sqrt{2}$; $7x^6-\frac{3}{2}x^4+4x^2+x-8$ etc are called Polynomials.

General form of a Polynomial: An algebraic

Expression of the form $P(x) = a_0+a_1+a_2+a_2x^2+a_3x^3+$- - - - - - - - - - - -$+a_nx^n$ where

$a_n \neq 0$ is called a Polynomial in variable x of degree n.

Here a_0, a_1,a_2,a_3- - - - - - - -,a_n are real numbers and each power of x is a non-negative Integer.

The highest Power in x of the Polynomial is called the degree of the Polynomial.

The representation of the Polynomial in the ascending or descending order of the powers of the polynomial is called the standard form of the Polynomial.

It is the degree of the Polynomial that classifies Polynomials. The different types of the Polynomial are as follows

- A Polynomial of degree Zero is called a Constant Polynomial.
- A Polynomial P(x) =ax+b of degree 1 is called a linear Polynomial.
- A Polynomial $P(x) = ax^2+bx+c$ of degree 2 is called quadratic Polynomial.
- A Polynomial $P(x) = ax^3+bx^2+cx+d$ of degree 3 is called a cubic Polynomial.
- A Polynomial $P(x) = ax^4+bx^3+cx^2+dx+e$ of degree 4 is called a bi-quadratic Polynomial

We also classify Polynomials on the basis of number of terms

1. **Zero Polynomial**: if all the terms of a Polynomial are zero`s e.g. $0.x^n+0.x^{n-1}+0.x^{n-2}+0.x^{n-3}+$- - -0.x+0 then it is called a zero Polynomial. The degree of a zero Polynomial is not defined.

2. **Monomial**: A Polynomial of one single term is called a monomial e.g. 2x, $\frac{3}{4}x^2y$; $2x^2yz$ etc.

3. **Binomial**: A Polynomial having two terms is called a binomial e.g. $2x+1$; $x+y$; $3x^2y+z$; $3xy+2x$ etc.

4. **Trinomial**: A Polynomial having three terms is called a Trinomial e.g. $x+y+z$; $2x-3y-4$; $2x^2y+3xy^2-4xy$ etc.

5. **Quadrinomial:** A Polynomial having four terms is called a quadrinomial e.g. $2x+3y+5z+6$; $3x-y+5z-xy$ etc. And so on.

In general a Polynomial means having many terms say 5, 6,7 ,10 etc

Consider the Polynomial, $P(x) = x^2-3x-4$. Putting x=2 in the Polynomial we get $P(2) = 2^2-3(2)-4 = 4-6-4 = -6$ the value " -6" is obtained by replacing x by 2 in x^2-3x-4 is the value of Polynomial x^2-3x-4 at x=2.

- If P(x) is a Polynomial in x and if k is any real number then the value obtained by replacing x by k in P(x) is called the value of p (x) at x=k and is denoted by P (k).

Again consider $P(x) = x^2-3x-4$.

$$P(-1) = (-1)^2-3(-1)-4 = 1+3-4 = 0$$

$P(4) = 4^2-3(4)-4 = 16-12-4 = 16-16 = 0$ Here -1 and 4 are called the Zero`s of the Polynomial.

Zero of Polynomial: A real number k is said to be a zero of a Polynomial P(x) if P (k) = 0

- Geometrically, the zeros of a Polynomial P(x) are precisely the x coordinates of of the points, where the graph of y = P(x) intersects the x axis.
- A quadratic Polynomial can have at most two zeros and a cubic Polynomial can have at most three zeros.
- In general a Polynomial of degree 'n' has at most 'n' zeros.

Relation between zeros and co-efficient of Polynomial

1. For Linear Polynomial $ax+b$; $a\neq0$

$x = -\frac{b}{a}$ here zero of a linear Polynomial is $K = -\frac{b}{a} = -\frac{\text{constant}}{\text{co-efficient of } x}$

2. For quadratic Polynomial : ax^2+bx+c ; $a\neq0$

If α, β are zeros of this quadratic Polynomial.

Then the sum of zeros = $\alpha + \beta = \frac{\text{co-efficient of } x}{\text{co-effiecient } x^2} = -\frac{b}{a}$

Product of zeros= $\alpha\beta$= constant term/co-efficient $x^2 = \frac{c}{a}$

3. Cubic Polynomial : ax^3+bx^2+cx+d ; $a\neq0$

   Let α, β, Y be the zeros of cubic Polynomial

   Then sum of zeros = $\alpha+\beta+\gamma = -\frac{\text{co-efficient of } x^2}{\text{cofficient of } x^3} = -\frac{b}{a}$

   Product of zeros taken two at a time = $\alpha\beta+\beta\gamma+\gamma\alpha = \frac{\text{co-efficient of } x}{\text{cofficient of } x^3} = \frac{c}{a}$

   Product of zeros = $\alpha\beta Y = \frac{\text{constant term}}{\text{cofficient of } x^3} = \frac{d}{a}$

<u>FORMATION OF POLYNOMIALS:</u>

- A quadratic Polynomial whose zeros are α and β is given by $P(x) = x^2-(\alpha+\beta)x+\alpha\beta$.

  i.e. $P(x) = x^2$-(sum of zeros)x+ Product of zeros

- A cubic Polynomial whose zeros are α ,β, γ is given by

  $P(x) = x^{3-}\ (\alpha+\beta+\gamma)x^2+(\alpha\beta+\beta\gamma+\gamma\alpha)x-\alpha\beta r$

- For more general form of a Polynomial development.

  Example: let the sum and product of zeros of a quadratic Polynomial be $\sqrt{2}; \frac{1}{3}$

Sol: let ax^2+bx+c be a quadratic Polynomial.

Let α, β, be its zeros

$\therefore$. Sum of zeros = $\alpha+\beta = -b/a = \sqrt{2} = \frac{3\sqrt{2}}{3}$ Making denominators same

Product of zeros $=\alpha\beta = \frac{c}{a} = \frac{1}{3} = \frac{1}{3}$

Setting a= 3, the common denominator of the above two equations

$\therefore -b = 3\sqrt{2} \Rightarrow b = -3\sqrt{2}$ and c=1

So one quadratic Polynomial which fits the given condition is $3x^2-3\sqrt{2}\,x+1$

But we can set a= any multiple of 3 say = 3K, we can check other quadratic Polynomials fitting above conditions will be of form $K(3x^2-3\sqrt{2}x+1)$

Similar relation holds between the zeros of a cubic Polynomial and its co-efficient and for other higher degree Polynomials as well.

The zeros of a quadratic Polynomial $ax^2+bx+c;\ a\neq0$ are precisely the x - coordinates of the points where the parabola representing $y= ax^2+bx+c$ intersects the x-axis

In fact, for any quadratic polynomial ax^2+bx+c , $a\neq0$ the graph of the corresponding equation $y = ax^2+bx+c$ has one of the two shapes either open upwards like ∪ or open downwards like ∩ depending whether $a>0$ or $a<0$ these curves are called Parabolas.

(For graphs see the text book)

Extreme values of quadratic Polynomials:

Let ax^2+bx+c be a quadratic Polynomial: $a\neq0$

Then $ax^2+bx+c= a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right) = a\left\{\left(x+\frac{b}{2a}\right)^2+\left(\frac{c}{a}-\frac{b^2}{4a^2}\right)\right\} = a\left\{\left(x\frac{+b}{2a}\right)^2+\frac{4ac-b^2}{4a}\right\}$

<u>**Case I**</u> when $a>0$ then $ax^2+bx+c \geq a\ \frac{4ac-b^2}{4a^2} = \frac{4ac-b^2}{4a}$

$\therefore$ Minimum value of ax^2+bx+c is $\frac{4ac-b^2}{4a}$

Which occurs when $x = -\frac{b}{2a}$

<u>**Case II**</u> When $a<0$, then $ax^2+bx+c \leq \frac{4ac-b^2}{4a}$

Example: Let $P(x) = x^2-5x+6$ Here $a=1$, $b= -5$ $c = 6$

$a > 0$, Minimum value of ax^2+bx+c is $\frac{4ac-b^2}{4a}$

$\therefore$ Minimum value of x^2-5x+6 is $\frac{4\times1\times6-(-5)^2}{4\times1} = \frac{24-25}{4} = -\frac{1}{4}$

Which occurs at = $-\frac{b}{2a} = -\left(\frac{-5}{2(1)}\right) = \frac{5}{2}$

<u>Example 2</u> Let P(x) = $-x^2+5x+6$ be the quadratic Polynomial. Here a < 0; a = −1, b= 5; c= 6

The max. Value = $\frac{4ac-b^2}{4a} = \frac{4(-1)6-5^2}{4(-1)} = \frac{-24-25}{4} = \frac{-49}{-4} = \frac{49}{4}$

Which occurs at x= $-\frac{b}{2a} = -\frac{5}{2(-1)} = \frac{5}{2}$

DIVISION ALGORITHM FOR POLYNOMIALS:

If p(x) and g(x) are two polynomials with g(x) ≠ 0, then we can find polynomials q(x) and r(x) such that

P(x) = g(x) × q(x) +r(x)

Where r(x) = 0 or degree of r(x) ∠ degree of g(x).

- If r(x) = 0 then g(x) is a factor of p(x)

•Dividend = Divisor × Quotient + Reminder

Factorization:

A. By splitting middle term

Example 1: Let P(x) = $x^2-5x + 6$

Sol: P(x) = $x^2-2x-3x+6$

= (x) (x−2) −3 (x−2)

∴ P(x) = (x−2) (x−3)

$2\,|\,\underline{6}$
$\quad 3$

(−2) + (−3) = −5

and

(−2) × (−3) = 6

Example 2: Let $P(x) = x^2+5x+6$

$P(x) = x^2+2x+3x+6$

$= x(x+2) + 3(x+2)$

Hence $P(x) = (x+2)(x+3)$

$2 \mid 6$, 3

$2+3 = 5$

$2\times3 = 6$

Example 3: Let $P(x) = 4x^2+8x+3$

Sol: $P(x) = 4x^2+2x+6x+3$

$= 2x(2x+1) +3(2x+1)$

$P(x) = (2x+1)(2x+3)$

$4 \times 3 = 12$

$2 \mid 12$, 3

$2 + 6 = 8$

$2 \times 6 = 12$

B. Factorization using Reminder Theorem:

Statement: if the polynomial f(x) is divided by $x-a$, then the reminder is equal to f(a).

If $f(a) = 0$, the reminder when f(x) is divided by $x-a$ is zero . Then f(x) is divisible by $x-a$ i, e. $(x-a)$ is a factor of f(x).

Example: Let polynomial $f(x) = x^2-4x+4$

Here $f(2) = 2^2-4(2)+4= 4-8+4=0$

$\therefore$ $X-2$ is a factor of x^2-4x+4.

The other factor of the expression can be found out .similarly or by taking out $x-2$ as a factor of expression (long division method can be used)

- It is wise to explain reminder theorem once more in order to explain the procedure of factorizing a polynomial of degree more than or equal to 3.
- For a quadratic polynomial $P(x)=ax^2+bx+c$; $a\neq0$ $D= b^2-4ac$

a) If D= 0; P(x) is a perfect square, having two equal factors e.g. x^2-4x+4

Here $D= (-4)^2-4(1)(4) = 16-16= 0$

We write $x^2-4x+4 = x^2-4x+4 = x^2-2\times2\times x+2^2 = (x-2)^2$

$= (x-2)(x-2)$

b) If D> 0 (+ve) ; P(x) will have two different factors

c) If D< 0 (− ve); No Linear factors possible for P(x)

e.g. x^2+x+1; Here $D= 1^2-4(1)(1) = -3<0$

=> x^2+x+1 cannot be factorised into linear factors.

- If $P(x) = ax^2+bx+c$ and D=0

  Then $ax^2+bx+c = a\left(x\frac{+b}{2a}\right)^2$

VERY SHORT ANSWER TYPE QUESTIONSS

Q1:- if one of the zero of quadratic polynomial x^2+3x+K is 2, then the value of K is

(a) 10 (b) −10 (c) 5 (d) −5

Q2:- A quadratic polynomial whose zeros are −3 and 4 is

(a) X^2-x+12 (b) x^2+x+12 (c) x^2-x-12 (d) $x^2-7x-12$

Q3 The relation between the zeros and co-efficient of the quadratic polynomial as ax^2+bx+c is

(a) $\alpha + \beta = \frac{c}{a}$ (b) $\alpha+\beta = \frac{b}{a}$ (c) $\alpha+\beta = \frac{-c}{a}$ (d) $\alpha+\beta= \frac{-b}{a}$

Q4 The zeros of the polynomial $x^2+7x+10$ are

(a) 2 and 5 (b) −2 and 5 (c) −2 and −5 (d) 2 and −5

Q:-5 The relationship between zeros and co-efficient of the quadratic polynomial ax^2+bx+c is

(a) α× $= \frac{c}{a}$ (b) α × $= \frac{-b}{a}$ (c) αβ= $\frac{-c}{a}$ (d) αβ= $\frac{b}{a}$

Q6 α, β are the zeros of the polynomial $f(x) = x^2+x+1$ then $\frac{1}{\alpha}+\frac{1}{\beta} =$

(a) 0 (b) 1 (c) −1 (d) None of these

Q7 If the sum of zeros of the polynomial $f(x) = 2x^2-3kx^2+4x-5$ is 6, then value of k is

(a) 2 (b) 4 (c) −2 (d) −4

Q8:- The zeros of a polynomial P(x) are precisely the x−coordinates of the points, where the graph of y=P(x) intersects the

(a) x−axis (b) y-axis (c) origin (d) none of these.

Q9:- A quadratic polynomial can have at most -------Zeros

(a) 0 (b) 1 (c) 2 (d) 3

Q10:- A polynomial of degree n has at most ----- zeros

(a) 0 (b) n+1 (c) n (d) n−1

Q11:- which of the following is not a polynomial?

(a) $\sqrt{3}\,x^2-2\sqrt{3}\,x+3$ (b) $\frac{3}{2}x^3-5x^2-\frac{1}{\sqrt{2}}x-1$ (c) $x+\frac{1}{x}$ (d) $5x^2-3x+\sqrt{2}$

Q12:- On dividing x^3+3x^2+3x+1 by x+1 the reminder is

(a) 0 (b) 1 (c) 2 (d) 3

Q13:- a quadratic polynomial whose sum and product of zeros are −5 and 6 is

(a) x^2-5x-6 (b) x^2+5x-6 (c) x^2+5x+6 (d) None of these

Q14:- which are the zeros of $P(x)=x^2+3x-10$

(a) 5, −2 (b) −5, 2 (c) −5, −2 (d) none of these.

Q15:- A real number K is called a zero of polynomial f(x) then

(a) f(k) = −1 (b) f(k) = 1 (c) f(k)=0 (d) f(k)= −2

Q16:- which of the following is polynomial:

(a) $x^2+\frac{1}{x}$ (b) $2x^2-3\sqrt{x}+1$ (c) $x^2+x^{-2}+7$ (d) $3x^2-3x+1$

Q17:- if the sum of the zeros of the polynomial $3x^2-kx+6$ is 3 then the value of k is

(a) 3 (b) −3 (c) 6 (d) 9

S.A Type: 2 Marks each

Q1:- if $2x-1$ and $x+2$ are the length and breadth of a rectangle in centimetres. Find its area A(x).

Q2:- if $2x-1$ and $x+2$ are the length and breadth of a rectangle in centimetres. For what value of x it will become a square?

Q3 :- if $A(x) = 2x^2-7x+6$ be the area function of a rectangle .find its length and breadth.

Q4 :- Fill in the blanks:

A quadratic polynomial ax^2+bx+c can be factorised into

(i) two ---------- if $D>0$

(ii) Two ------------ if $D=0$

(iii) Cannot have two linear factors if D is --------

Q5:- if $P(x) = 3x^3-2x^2+6x-5$; Find P (2)

Q6:- Draw the graph of polynomial $f(x) = x^2-2x-8$ Read the zeros of polynomial from the graph.

Q7:- Draw the graph of polynomial $f(x) = -4x^2+4x-1$

Read the zeros from the graph.

Q8:- Find the quadratic Polynomial whose zeros are

$\frac{3-\sqrt{3}}{5}$ and $\frac{3+\sqrt{3}}{5}$

Q9:- Match column 'A' with column 'B'

A	B
(i) P(x) be polynomial, g(x) the Divisor q(x) the quotient and r(x) The reminder then division algorithm Is written as	(a) P(x) and g(x) are of same degree
(ii) Remainder r(x) is zero if	(b) less than degree of g(x)
(iii) If P(x) is divided by g(x) and degree of quotient is zero Relation between P(x) and g(x)	(c) P(x) = g(x) × q(x) + r(x)
(IV) Degree of r(x) is always	(d) g(x) is factor of P(x)

Q10:- If 2 and -3 are zeros of the polynomial $x^2+(a+1)x-b$. Then find the values of a and b.

Q11:- On dividing x^3-3x^2+x+2 by a polynomial g(x) , the quotient and remainder were $x-2$ and $-2x+4$ respectively .Find g(x).

Q12:- If the product of zeros of a polynomial ax^2-6x-6 is 4. Find the values of 'a' .

Q13:- Write a quadratic polynomial, sum of whose zeros are $2\sqrt{3}$ and their product is 2.

Q14:- Find the sum and product of zeros of $P(x) = 2(x^2-3)+x$.

L.A Type 3 marks each

Q1. Find the zeros of the quadratic polynomial $6x^2-7x-3$ and verify the relationship between the zeros and the co-efficient.

Q2. Find a quadratic polynomial, the sum and product of whose zeros are $\sqrt{2}$ and $-\frac{3}{2}$

Q3. If one zero of the quadratic polynomial x^2+3x+k is 2, then find the value of k.

Q4. Given that one of the zeros of the cubic polynomial ax^3+bx^2+cx+d is zero, then find the product of the other two zeros.

Q5. If one of the zeros of the cubic polynomial x^3+ax^2+bx+c is -1, then find the product of the other two zeros.

Q6. If one of the zero`s of the quadratic polynomial $(k-1)x^2+kx+1$ is -3, then find value

of k.

Q7. If the zero`s of the quadratic polynomial $x^2+ (a+1) x+ b$ are 2 and -3, then find the values of 'a' and 'b'

Q8. If α,β are zero`s of the quadratic polynomial $x^2-(k+6)x+2(2k-1)$. Find value of k if $\alpha+\beta=\frac{1}{2}\alpha\beta$

Q9. If polynomial $6x^4+8x^3-5x^2+ax+b$ is exactly divisible by the polynomial $2x^2-5$, then find the values of a and b

Q10. Find a cubic polynomial whose zero`s are 3, $\frac{1}{2}$ and -1.

Q11. Verify that 5, -2 and $\frac{1}{3}$ are the zero`s of the polynomial $P(x) =3x^2-2x^2-5x+6$.

Q12. Find the quotient and remainder when $4x^3+2x^2+5x-6$ is divided by $2x^2+3x+1$

Q13. On dividing x^4-5x+6 by a polynomial g(x), the quotient and remainder were $-x-2$ and $-5x+10$ respectively. Find g(x).

Q14. Given that $\sqrt{2}$ is zero of the cubic polynomial $6x^3+\sqrt{2}\,x^2-10x-4\sqrt{2}$ .find its other two zero`s.

Q15. If α,β are the zeros of polynomial $f(x) = 6x^2 + x - 2$ find the value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

Q16. Find value of k so that x^2+2x+k is a factor of $2x^4+x^3-14x^2+5x+6$.

VERY LONG ANSWER TYPE (4 marks each)

Q1. If the polynomial $x^4-6x^3+16x^2-25x+10$ is divided by another polynomial x^2-2x+k, then remainder comes out to be x+a. Find k and a.

Q2 If α,β are the zero`s of the quadratic polynomial $P(x)=x^2-3x-2$,then find a quadratic polynomial whose zero`s are $\frac{1}{2\alpha+\beta}$ and $\frac{1}{2\beta+\alpha}$.

Q3 If α,β are the zero`s of quadratic polynomial $P(x)=2x^2-5x+7$, then find a quadratic polynomial whose zero`s are $2\alpha+3\beta$ and $3\alpha+2\beta$.

Q4. If α,β are the zero`s of polynomial $f(x)=x^2-P(x+1)-c$, show that $(\alpha+1)(\beta+1)=1-c$

Q5. What must be subtracted from $8x^4+4x^3-2x^2+7x-8$. So that the resulting polynomial is exactly divisible by $4x^2 + 3x + 2$.

Q6. What must be added to the polynomial $4x^4+2x^3-2x^2+x-1$ so that the resulting polynomial is exactly divisible by x^2+2x-3

Q7. Find all the zero`s of the polynomial $x^4-6x^3-26x^2+138x-35$, if two of its zero`sare $2+\sqrt{3}$ and $2-\sqrt{3}$.

Q8. Find values of a and b so that $x^4+x^3-8x^2+ax+b$ is divisible by x^2+1.

Q9. If the polynomial $f(x) = x^4-6x^3+16x^2-25x+10$ is divided by another polynomial x^2-2x+k, the remainder comes out to be $x+a$ find k and a .

Q10. If α and β are the zero`s of the quadratic polynomial $f(x) =x^2-2x-8$, then find the value of (i) $\alpha-\beta$ (ii) $\alpha^2+\beta^2$.

Q11. Divide $3x^2-x^3-3x+5$ by $x-1-x^2$ and verify the division algorithm.

Q12. Find the zero`s of the quadratic polynomial $f(x) =abx^2+(b^2-ac)x-bc$ and verify the relationship between the zero`s and its co-efficients.

Q13. Find a quadratic polynomial, the sum and product of whose zero`s are $\sqrt{2}$ and $\frac{1}{3}$ respectively. Also find its zero`s

ANSWERS

VERY SHORT ANSWER TYPE

Q1 (b) Q2 (c) Q3 (d) Q4 (c) Q5 (a) Q6 (c) Q7 (b) Q8 (a) Q9 (c) Q10 (c)

Q11 (c) Q12 (a) Q13 (c) Q14 (b) Q15 (c) Q16 (d) Q17 (d)

SHORT ANSWER TYPE.

Q1 A (x)=$2x^2+3x-2$ Q2 x=2 Q3 L=2x-3 ; B=x−2

Q4. (i) Distinct real linear factors (ii) Equal real linear factors

(iii) D is less than zero Q5 23 Q6 4 and −2

Q7. $\frac{1}{2}, \frac{1}{2}$ Q8. $25x^2-30x+6$ Q9 (i) (c) (ii) (d) (iii) (a) (iv) (b)

Q10. a=0; b= −6 Q11. X^2-x+1 Q12. a=$-\frac{3}{2}$ Q13. $X^2-2\sqrt{3}\,x+2$ Q14. Sum= $-\frac{1}{2}$

product= −3

LONG ANSWER TYPE.

Q1 $\frac{3}{2}$ and $-\frac{1}{3}$ Q2. $2x^2-2\sqrt{2}\,x-3$ Q3. K= −10 Q4. $\frac{c}{a}$ Q5. a-b-1 Q6. K=$^4/_3$

Q7. a=0; b= -6 Q8. K=7 Q9. a= -20; b= -25 Q10. $2x^3-5x^2-4x+3$ Q12. 2x-2, R=9x-4

Q13. g(x) = -x+2 Q14 $-\frac{\sqrt{2}}{2}$ and $-2\frac{\sqrt{2}}{2}$ Q15. $\frac{-25}{12}$ Q16. K= $-\frac{27}{7}$

VERY LONG ANSWER TYPE

Q1. K=5 and a=35 Q2 $16x^2-9x+1$ Q3. $2x^2-25x+82$ Q5. 12x-2 Q6. 61x-65

Q7. -5 and 7 Q8 a=1 and b=7 Q9. K=5 and a= -5 Q10. (i) 6 (ii) 20

Q11. $-x^3+3x^2-3x+5 = (-x^2+x-1)(x-2)$ Q12 $\frac{c}{b}$ and $\frac{-b}{a}$ Q13. $3x^2-3\sqrt{2}\,x+1$; Zero`s are $\frac{3\sqrt{2} \pm \sqrt{6}}{6}$