CORORDINATE GEOMETRY

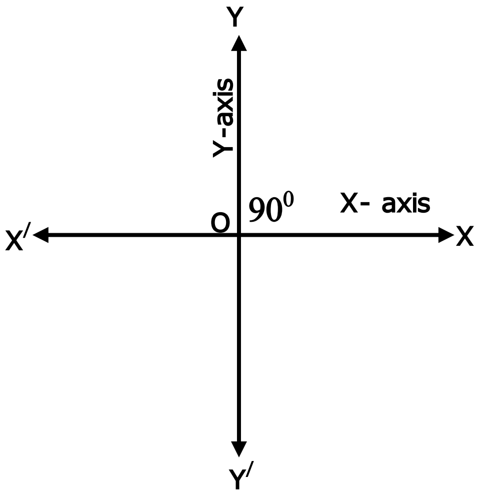
CLASS – 10^{TH} CHAPTER – 7

7. 1 Introduction:

Coordinate Geometry is that branch of geometry which defines the position of a point in a plane by a pair of algebraic numbers. It is also called ALGEBRAIC GEOMETRY** OR **ANALYTICAL GEOMETRY.

7. 2 Rectangular Axis and Origin:

Let $X^{/}OX$ and $Y^{/}OY$ be two perpendicular straight lines intersecting at the point "O". Then



i. $X^{/}OX$ is called the *axis of "x"* or *the "x-axis"*.

ii. $Y^{/}OY$ is called the *axis of "y"* or the "*y – axis"*.

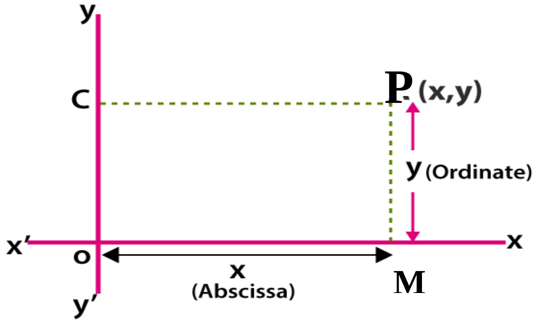
iii. Both $X^{/}OX$ and $Y^{/}OY$ taken together, in this very order, are called the "*Rectangular axes"* or the "*axes of Co-ordinates"* or *the "Coordinate axes"* or simply the *"axes"*.

(**Note – 1 :** *They are called Rectangular axes because the angle between them is a Right angle.)*

iv. Their point of intersection "O" is called the origin.

7. 3 Cartesian Co-ordinates of a point:

Let $X^{/}OX$ and $Y^{/}OY$ be two perpendicular lines intersecting at the point "O". Let "P" be any point in the plane of the axes. From "P", draw PM perpendicular $X^{/}OX$, then



a) "OM" is called the *"x-ordinate"* or "*abscissa of P"* and is denoted by "x"

b) "MP" is called the *"y-coordinate"* or "*Ordinate of "P"* and is *denoted by "y".*

c) The numbers "x" and "y" are called the *Cartesian rectangular Coordinate* or simply the "*Coordinate of P"*, represented by *P(x,y).*

(**Note – 2 :** *In this symbolic representation i.e. P(x,y), the "Abscissa is always written first and separated from the Ordinate (at second place) by a comma.*

Remember:

1. Abscissa is the perpendiculars distanc from "y – axis".
2. Ordinate is the distance of a point from "x axis".
3. Abscissa is +ve to the right to the "y-axis" and –ve to the left of "y-axis".
4. Ordinate is +ve above "x-axis" and –ve below "x-axis".
5. Abscissa of any point on "y – axis" is zero
6. Ordinate of any point on "x-axis" is zero.
7. Corodinates of the origin are (0,0).

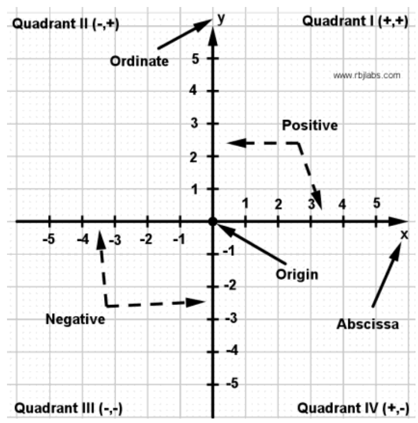
Note – 3: *The two axis divide the plane into four regions called the Quadrants. The signs of the Co-ordiantes in different Quardants are:*

a) (+,+) ⟶ both abscissa and Ordinate are +ve in first quadrant.

b) (-, +) ⟶ in second quadrant.

c) (-, -) ⟶ in third quadrant.

d) (+, -) ⟶ in fourth quadrant.



7. 4 Distance between two Points: *(Distance Formula)*

Let P(x_1, y_2) and Q (x_1, y_2) be the given points. Draw PL and QM perpendicular OX and PR perpendicular QM. Then

PR = LM = OM – OL = $x_2 - x_1$

RQ = MQ – MR = MQ – LP = $y_2 - y_1$

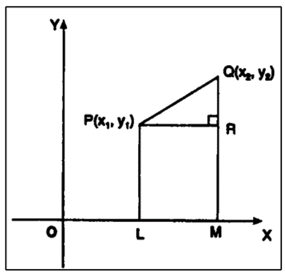
In right angled Δ PRQ

BY Pythagoras Theorem

$PQ^2 = PR^2 + RQ^2$

$= (x_2 - x_1)^2 + (y_2 - y_1)^2$

PQ $= \sqrt{(x_1 - x_2)^2 + (y_2 - y_1)^2}$



Corollary: Distance of a point (x, y) from the origin (0, 0) = $\sqrt{x^2 + y^2}$

Note – 4: *When three points are given and it is required to prove that they form:*

a) an Isosceless triangle, show that two of its sides are equal

b) an equilateral triangle, show that its three sides are equal.

c) a right angled triangle, show that the square of one side is equal to the sum of the square of other two sides.

d) They are collinear, show that the sum of the distances between two points is equal to the distances between the first point and the third point.

Note – 5: *When four points are given and it is required to prove that they form a;*

a) Square, show that all sides are equal and diagonals are equal.

b) Rhombus, show that all sides are equal and diagonals are unequal.

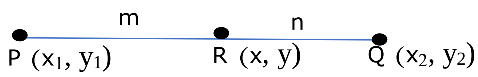
c) Rectangle, show that the opposite sides are equal and diagonals are also equal.

d) Parallelogram, show that the opposite sides are equal.

7. 5 Section Formula:

Let R(x,y) be the point which divides the joining of P(x_1, y_1) and Q = (x_2, y_2) in the ratio m:n internally. The "R" lies between "P" and "Q" and have Coordinates as

$$\mathbf{R(x,y) = R}\left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$



i.e. *x-Coordinate of R is* $\frac{mx_2 + nx_1}{m+n}$ *and*

y-coordiante of R is $\frac{my_2 + ny_1}{m+n}$

This is known as ***Section Formula.***

Note: If we have to find the ratio in which "R" divides "PQ", it is convenient to take the ratio K:1 instead of m:n

Corollary: Mid Point Formula: If "R" is the mid – point of "PQ", then "m = n" i.e. the ratio is 1:1.

Therefore, Co-ordinates of the mid-point of the line – segement joining P(x_1, y_1) and Q (x, y_2) are

$$R\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right) \quad \text{i.e}$$

$$R\left[\frac{\textit{Sum of absissae}}{2}, \frac{\textit{Sum of Ordinates}}{2}\right]$$

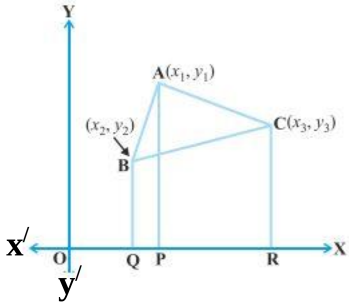
Remember:

1. The median is a line joining a vertex of the triangle to the middle point of the opposite side.
2. The point of intersection of the medians of a triangle is called the "*Centroid*" of the triangle.
3. The Centroid of a triangle divides each median in the ratio 2:1, 2 always being on the sides of the vertex.
4. The Co-ordiantes of the Centroid of a triangle whose vertices are (x_1, y_1), (x_2,y_2) and (x_3, y_3) are:

$$\left\{ \frac{x_1+x_2+x_3}{3} \quad , \quad \frac{y_1+y_2+y_3}{3} \right\}$$

7. 6 Area of a Triangle:

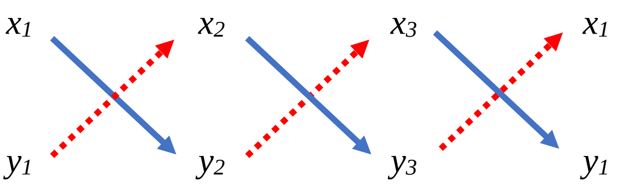
If A (x_1, y_1), B (x_2, y_2) and C(x_3, y_3) are the vertices of a triangle ABC, then the area of triangle is given by



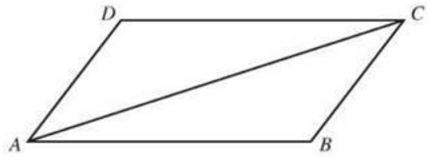
$$\text{Area of } \Delta \text{ ABC} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

Note – 6:

1. *If during calculations, area comes out to be negative, then we take it absolute value.*
2. *If we have to find some conditions, where area of a triangle is given, then we take both signs under consideration.*
3. *We can find area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) by using following diagram.*



We multiply the terms connected by arrows, for bold arrow, we put plus sign and for dotted arrow, we put negative sign.



Area of Triangle = $\frac{1}{2}$ **{ $x_1y_2 + x_2y_3 + x_3y_1 - x_2y_1 - x_3y_2 - x_1y_3$ }**

4. *If we have to find the area of a quadrilateral ABCD, we can draw one diagonal and divide the quadrilaterals into two triangles and then apply formula for finding area of a triangle.*

Area (Quad. ABCD) = ar $(\Delta\, \boldsymbol{ABC})$ ***+ ar*** $(\Delta\, \boldsymbol{BCD})$

7. 7 Condition for Collinearity:

A (x_1, y_1), B (x_2, y_2) and C(x_3, y_3) are collinear (i.e. the three points lie on same straight line). If the area of triangle formed by these vertices is zero.

i.e. Area of Triangle = 0, if A, B, C are collinear.

1 MARK QUESTIONS

Choose the correct Answer

Q1 The distance of the point P (2, 3) from the x-axis is

a) 2 b) 3 c) 1 d) 5

Q.2 The distance between the points A (0,6) and B (0, -2) is

a) 6 b) 8 c) 4 d) 2

Q.3 The distance of the point P (-6, 8) from the origin is

a) 8 b) $2\sqrt{7}$ c) 10 d) 6

Q.4 The distance between the points (0, 5) and (-5, 0) is

a) 5 b) $5\sqrt{2}$ c) $2\sqrt{5}$ d) 10

Q.5 AOBC is a rectangle whose three vertices are vertices A (0, 3), O (0, 0) and B (5,0). The length of its diagonal is

a) 5 b) 3 c) $3\sqrt{4}$ d) 4

Q.6 The perimeter of a triangle with vertices (0,4), (0,0) and (3,0) is

a) 5 b) 12 c) 11 d) $7\sqrt{5}$

Q.7 The area of a triangle with vertices A (3, 0), B (7, 0) and C (8, 4) is 28 (T/F)

Q.8 The points (-4, 0), (4, 0), (0, 3) are vertices of an Isosceles Triangle. (T/F)

Q.9 If the distance between the point (2,-2) and (-1, x) is 5, one of the value of x is 1 (T/F)

Q.10 The mid points of the line segment joining the points A (- 2, 8) and B (-6, -4) is (- 4, 2) (T/F)

Q.11 The points A (9, 0), B (9, 6), C (-9, 6) and D (-9, 0) are the vertices of a Rhombus (T/F)

Q.12 The point which divides the line segment joining the points (7,-6) and (3, 4) in the ratio 1:2 internally lies in the

a) 1 - Quadrant b) II – Quadrant c) III – Quadrant
d) IV – Quadrant.

Q.13 The point which lies on the perpendicular bisector of the line segment joining the point A (-2,-5) and B (2, 5) is

a) (0, 0) b) (0, 2) c) (2, 0) d) (-2, 0)

Q.14 The fourth vertex D of a parallelogram ABCD whose three vertices are A (-2, 3), B (6, 7) and C (8, 3) is

a) (-2, 3) b) (0,-1) c) (-1, 0) d) (1, 0)

Q.15 If the point P (2, 1) lies on the line segment joining points A (4, 2) and B (8, 4), then

a) AP = $\frac{1}{3}$ AB b) AP = PB c) PB = $\frac{1}{3}$ AB d) AP = $\frac{1}{2}$ AB

Q.16 If P ($\frac{a}{3}$, 4) is the midpoint of the line segment joining the point Q (-6, 5) and R (-2, 3), then the value of "a" is

a) -4 b) -12 c) 12 d) -6

Q.17 Line intersects the y-axis and x-axis at the points P and Q respectively, if (2,-5) is the midpoint of PQ, then the coordinates P and Q are respectively

a) (0,-5) and (2, 0) b) (0, 10) and (-4, 0) c) (0, 4) & (-10, 0)
d) (0,10) and 4,0)

Q.18 The area of a triangle with vertices (a, b + c), (b, c + a) and (a, b + c) is

a) (a+ b + c) ^2 b) 0 c) (a + b+ c) d) ab

Q.19 If the distance between the points (4,p) and (1,0) is 5, then the value of p is

a) 4 only b) $\pm$ 4 c) -4 only d) 0

Q.20 If the points A (1, 2), O (0,0) and C (a, b) are collinear, then

a) a = b b) a = 2b c) 2a = b d) a = -b

Q.21 If P (1, 2), Q (4, 6), R (5, 7) and S (a, b) are the vertices of a parallelogram PQRS then a =……………., b =…………….

Q.22 There are / is ……………. number of points on x-axis which are at a distance of 2 units from (2,4)

Q.23 The distance of the points (h, k) from x-axis is …………….

Q.24 ……………. is The area of the triangle with vertices at the points (a, b+ c) (b, c+ a) and (c, a+ b)

Q.25 The distance of A (5, -12) from the origin is …………

Answers

Q1.	(b)	Q.2	(b)	Q.3	(c)	Q.4	(b)	Q.5	(c)	Q6	(b)
Q.7	(F)	Q.8	(T)	Q.9	(F)	Q.10	(T)	Q.11	(F)	Q.12	(d)
Q.13	(a)	Q.14	(b)	Q.15	(d)	Q.16	(b)	Q.17	(d)	Q.18	(b)
Q.19	(b)	Q.20	(c)	Q.21	a=2, b=3		Q.22	(0)	Q.23	$\lvert K \rvert$	
Q.24	0	Q.25	(13)								

Short Answer Type Questions

Q.1 If A (6, -1), B(1,2) and C (K,3) are three points such that AB = BC. Find the value of K.

Q.2 The distance between the points P(a Sin θ, a cos θ) and Q (a cos θ,- a sin θ).

Q.3 Check whether the points (1,5), (2, 3) and (-2, -11) are collinear or not.

Q.4 Find a relation between "x" and "y", if the points (x, y), (1, 2) and (7, 0) are collinear.

Q.5 Find the point on x – axis which is equidistance from (2, -5) and (9, 1).

Q.6 Find the point on y – axis, each of which is at a distance of 13 units from the point (-5, 7).

Q.7 Show the point on A(1,2), B(5,4), C (3,8), D (-1, 6) are vertices of a square.

Q.8 If two vertices of an equilateral triangle are (0, 0) and (3,0), find the third vertex.

Q.9 Find the coordinates of the mid point of the line segement joining the points A (3,0) and B (5,4).

Q.10 The mid point of the line segment joining A(2a, 4) and B (-2, 3b) is M (1,2a +1). Find the vlaues of a and b.

Q.11 In what ratio does the points P (2,5) divide the line segment joining A (8, 2) and B (-6, 9).

Q.12 Find the coordinates of the point of trisection of the line segement joining the points (4, -1) and (-2, -3).

Q.13 Find the lengths of the medians of the triangle where vertices are A (1,-1), B(0,4) and C (-5, 3).

Q.14 AB is a diameter of a circle with centre C (-1, 6). If the coordinates of A are (-7, 3). Find the coordinates of B.

Q.15 Find the coordinates of the point P which is three – fourth of the way from A (3, 1) to B (-2, 5).

Q.16 Find th centroid of a triangle ABC whose vertices are A(-1,0), B(5,2) and C (8,2)

Q.17 Determine if the points (1,5), (2,3) and (-2, -11) are collinear using area of triangle.

Q.18 (Ref. using area of triangle) find the value of K if the points (8,1), (K, -4) and C (2, -5) are collinear.

Q.19 If A (2, 1), B (6,0), C(5, -2) and D (-3, -1) are the vertices of a quadrilateral. Find the area of quaderilatral ABCD.

Q.20 Find the value of K for which the area formed by the triangle with vertices A (K,0), B (4,0) and C (0, 2) is 4 square units.

ANSWER

Q.1	$\sqrt{33}+1$	Q.2	$a\sqrt{2}$	Q.3	(Not) area of triangle $\neq 0$
Q.4	(x+3y – 7 = 0)	Q.5	(2, 0)	Q.6	(0, 19) or (0,-5)
Q.8	$(\frac{3}{2}, \frac{3\sqrt{3}}{2})$ or $(\frac{3}{2}, -\frac{3\sqrt{3}}{2})$	Q.9	(-1,2)	Q.10	a = 2, b = 2
Q.11	(3:4)	Q.12	(2, -5/3) and (0,-7/3)	Q.13	$\frac{\sqrt{130}}{2}, \sqrt{3}$
Q.14	(5,9)	Q.15	(6/7, 19/7)	Q.16	(4,0) Q.17 No
Q.18	(K=3)	Q.19	+ 15 square unit	Q.20	K = (0, 8)

Long Answer Type

Q.1 Prove that the points A (a, a), B (-a, - a) and C ($-\sqrt{3}$ a, $\sqrt{3}$ a) are the vertices of equilateral triangle. Calculate the area of this triangle.

Q.2 If the distance of P(x, y) from A (5, 1) and B (-1, 5) are equal. Prove that 3x = 3y

Q.3 If A, B and P are the points (-4, 3), (0, -2) and (α, β) respectively and P is equidistance from A and B. Show that $8 \alpha \neq 10 \beta + 21 = 0$

Q.4 If P (a, -11), Q (5, b), R (2, 15) and S (1, 1) are the vertices of a parallelogram PQRS. Find the value of "a and b".

Q.5 In what ratio is the line segment joining A (6, 3) and B (-2, -5) is divided by the x-axis. Also find the coordinates of the point of intersection of AB and the x – axis.

Q.6 The point P (-4, 1) divide the line segment joining the points A (2,-2) and B is the ratio 3: 5. Find the point B.

Q.7 Show that the points (3,-2), (5, 2) and (8, 8) are collinear by using Section formula.

Q.8 A (3, 2) and B (-2, 1) are two vertices of a triangle ABC whose centroid is G (5/3, – 1/3). Find the coordinates of the third vertex "C".

Q.9 Calculate the ratio in which the line joining the points A (6, 5) and B (4, -3) is divided by the line y = 2. Also find the coordinates of the point of intersection.

Q.10 Find the area of the triangle formed by joining the mid points of the sides of the triangle whose vertices are (0, -1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

ANSWERS

Q.1	$2\sqrt{3}\,a^2$	Q.4	(a=4, b =3)	Q.5	{(3:5, (3, 0)}	Q.6	(-14, 6)
Q.8	(4, - 4)	Q.9	{3:5 , (21/4, 0)}	Q.10	(1 Sq unit, 1:4)		