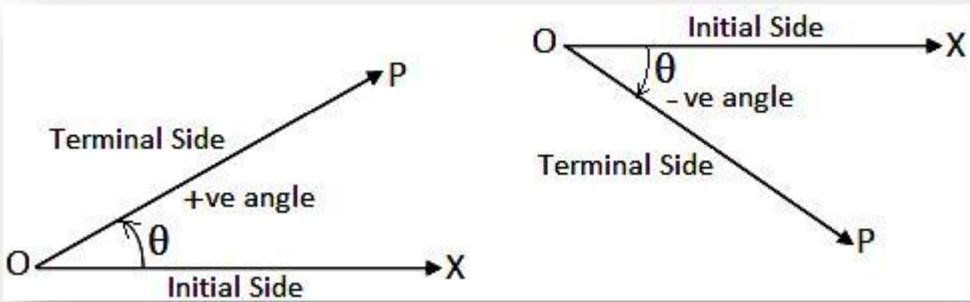
INTRODUCTION TO TRIGONOMETRY

Trigonometry originated as part of the study of triangle. The word ***Trigonometry*** is derived from the Greek Word "***Tri (means three) Gon (meaning sides)*** and ***metron (means measure)".*** In fact trigonometry means the measurement of three concerned figures, and the first definitions were in form of triangles. However, ***trigonometric functions*** can also be defined using the unit circle, ***a definition that makes them periodic or repeating.*** Many naturally occurring processes are also periodic, days and nights, seasons, water level in a tidal basin, the blood pressure in a heart, an alternating current and the position of air molecules, transmitting a musical note, all fluctuate regularly. Such a Phenomenon can be presented by Trigonometric functions.

The Sine and Cosine functions are commonly used to model periodic function phenomenon, such as sound and light waves. The position and velocity of harmonic oscillators, sunlight intensity and day length and average. Temperature variations throughout the year.

ANGLE: Angle is the figure obtained by the rotation of a given ray about its end point from its initial position to the terminal position.



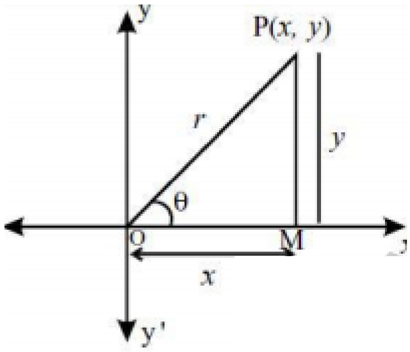
The measure of an angle is the amount of rotation from its initial position to the terminal position. If the *ray rotates in anticlockwise sense, the angle formed is taken positive.* If the *ray rotates in clockwise sense, the angle formed is taken negative.*

Remarks:

"OP" and "OX" are called arms of angle <POX and point "O" is called vertex of the angle.

Trigonometric Ratio (T – Ratio) of an acute angle of a Right Triangle:

In "XOY" – plane, let a revolving line "OP" starting from "OX", traces angle XOP= θ. From "P (x, y)" draw "PM perpendicular to "OX".



In right angled triangle OMP, OM = "x" (adjacent side), PM = "y" (opposite side): OP = "r" (hypotenuse).

1. Sin θ = $\frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{y}{x}$
2. Cos θ = $\frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{x}{r}$
3. Tan θ = $\frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{y}{x}$
4. Cosec θ = $\frac{\text{Hypotenuse}}{\text{Opposite}} = \frac{r}{y}$
5. Sec θ = $\frac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{r}{x}$
6. Cot θ = $\frac{\text{Adjacent side}}{\text{Opposite side}} = \frac{x}{y}$

Reciprocal Relations:

1	sinA.cosecA = 1	$\sin A = \frac{1}{\operatorname{cosec} A}$	$\operatorname{cosec} A = \frac{1}{\sin A}$
2	tanA.cotA = 1	$\tan A = \frac{1}{\cot A}$	$\cot A = \frac{1}{\tan A}$
3	cosA.secA = 1	$\cos A = \frac{1}{\sec A}$	$\sec A = \frac{1}{\cos A}$

Quotient Relations

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ *and* $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Remark 1 : Sin θ is read as the ***"Sine of angle θ"*** and it should never be interpreted as the product of ***"Sin" and "θ".***

Remark 2 : **Notation:** ***(Sin*** θ$)^2$ is written as ***Sin²*** θ ***(read "Sin square θ").*** Similarly ***(Sin θ)ⁿ*** is *written as* ***Sinⁿ*** θ (read ***Sin nth*** and ***power "θ"), "n"*** being ***positive integer.***

Note : **(Sin** θ$)^2$ should not be written as ***Sin*** θ^2 or as ***Sin²*** θ^2

Remark 3 : Trigonometric ratios depend only on the value of θ and are independent of the lengths of the sides of the right angle triangle.

Trigonometric Ratios of Complementary Angles:

- **Sin (90 – θ) = Cos θ** **Cos (90 – θ) = Sin θ**
- **Tan (90 – θ) = Cot θ** **Cot (90 – θ) = Tan θ**
- **Sec ((90 – θ) = Cosec θ** **Cosec (90 – θ) = Sec θ**

TRIGONOMETRIC RATIOS FOR ANGLE OF MEASURE 0^0, 30^0, 45^0, 60^0, AND 90^0 IN TABULAR FORM

$\theta =$	0^0	30^0	45^0	60^0	90^0
$\sin\theta =$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta =$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta =$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined
$\text{cosec}\ \theta =$	Undefined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta =$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Undefined
$\cot\theta =$	Undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Trigonometric Identities

An equation involving ***trigonometric ratios of an angle*** is said to be a ***trigonometric identity,*** if it is ***satisfied*** for all ***values of θ*** for which the given ***trigonometric ratios*** are ***defined***.

Identity – 1

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\cos^2\theta = 1 - \sin^2\theta$$

Identity – 2

$$\sec^2\theta - \tan^2\theta = 1$$

$$\tan^2\theta = \sec^2\theta - 1$$

Identity – 3

$$\text{Cosec}^2\theta = 1 + \cot^2\theta$$

$$\cot^2\theta = \text{Cosec}^2\theta - 1$$

Remark – 1

$$\frac{\sin^2\theta}{1-\cos\theta} = 1 + \cos\theta$$

Remark – 2

$$\frac{\cos^2\theta}{1-\sin\theta} = 1 + \sin\theta$$

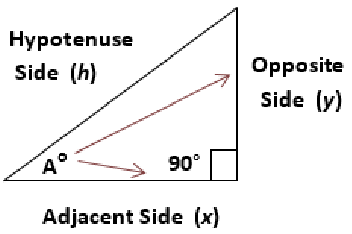
Remark – 3

$$\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$$

Remark –4

$$\text{cosec}\,\theta - \cot\theta = \frac{1}{\text{cosec}\theta + \cot\theta}$$

SOME TIPS

Right Triangle	SOH-CAH-TOA Method	Coordinate System Method
	SOH: $\text{sine}(A) = \sin(A) = \frac{\text{Opposite}}{\text{Hypotenuse}}$ CAH: $\text{cosine}(A) = \cos(A) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ TOA: $\text{tangent}(A) = \tan(A) = \frac{\text{Opposite}}{\text{Adjacent}}$	$\sin(A) = \frac{y}{h}$ $\cos(A) = \frac{x}{h}$ $\tan(A) = \frac{y}{x}$
	$\text{cosecant}(A) = \csc(A) = \frac{1}{\sin(A)} = \frac{\text{Hypotenuse}}{\text{Opposite}}$ $\text{secant}(A) = \sec(A) = \frac{1}{\cos(A)} = \frac{\text{Hypotenuse}}{\text{Adjacent}}$ $\text{cotangent}(A) = \cot(A) = \frac{1}{\tan(A)} = \frac{\text{Adjacent}}{\text{Opposite}}$	$\csc(A) = \frac{1}{\sin(A)} = \frac{h}{y}$ $\sec(A) = \frac{1}{\cos(A)} = \frac{h}{x}$ $\cot(A) = \frac{1}{\tan(A)} = \frac{x}{y}$

Some Trigonometric functions in terms of the other five

in terms of	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\sin\theta =$	$\sin\theta$	$\pm\sqrt{1-\cos^2\theta}$	$\pm\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\csc\theta}$	$\pm\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\pm\frac{1}{\sqrt{1+\cot^2\theta}}$
$\cos\theta =$	$\pm\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\pm\frac{1}{\sqrt{1+\tan^2\theta}}$	$\pm\frac{\sqrt{\csc^2\theta-1}}{\csc\theta}$	$\frac{1}{\sec\theta}$	$\pm\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$
$\tan\theta =$	$\pm\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\pm\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\pm\frac{1}{\sqrt{\csc^2\theta-1}}$	$\pm\sqrt{\sec^2\theta-1}$	$\frac{1}{\cot\theta}$
$\csc\theta =$	$\frac{1}{\sin\theta}$	$\pm\frac{1}{\sqrt{1-\cos^2\theta}}$	$\pm\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\csc\theta$	$\pm\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\pm\sqrt{1+\cot^2\theta}$
$\sec\theta =$	$\pm\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\pm\sqrt{1+\tan^2\theta}$	$\pm\frac{\csc\theta}{\sqrt{\csc^2\theta-1}}$	$\sec\theta$	$\pm\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$
$\cot\theta =$	$\pm\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\pm\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	$\pm\sqrt{\csc^2\theta-1}$	$\pm\frac{1}{\sqrt{\sec^2\theta-1}}$	$\cot\theta$

***Note:* Csc θ is same as Cosec θ**

Additional Formula

- ✓ ***Sin (A + B)*** = ***Sin A Cos B + Cos A Sin B***
- ✓ ***Sin (A – B)*** = ***Sin A Cos B – Cos A Sin B***
- ✓ ***Cos (A + B)*** = ***Cos A Cos B – Sin A Sin B***
- ✓ ***Cos (A – B)*** = ***Cos A Cos B + Sin A Sin B***
- ✓ ***Sin 2 A*** = ***2 Sin A Cos A***
- ✓ ***Tan 2 A*** = $\frac{2\,Tan\,A}{1 - Tan^2\,A}$

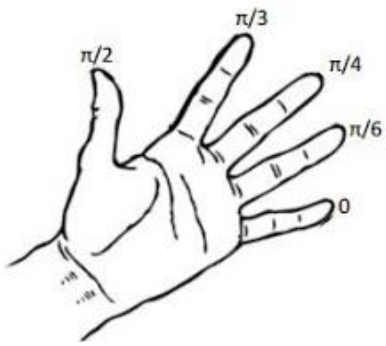
TRIGONOMETRIC HAND TRICK

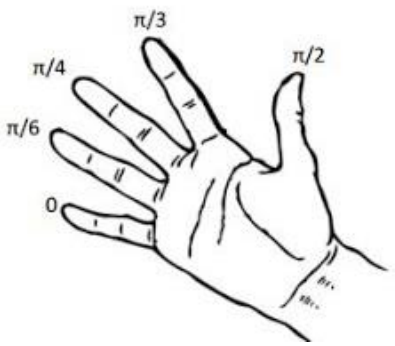
This is an easy way to remember the values of common values of trigonometric functions in the first quadrant. It's a lengthy explanation, but once you know this by heart, you can use this trick for all four quadrants. All you need is your non-dominant hand.

Step – 1 : Hold out your non-dominant hand.

Step – 2 : Assign" the following values to your fingers.

If your non-dominant hand is your left hand	**If your non-dominant hand is your right hand.**





Step – 3 : Find a trig problem. e.g. Cos $(\frac{\pi}{6})$

Step – 4 : Hold down the finger assigned for that angle.
For example: Hold down your ring finger for π/6

Step – 5 : Know the following Formulas

$$\sin\theta = \frac{\sqrt{bottom\ fingers}}{2} \qquad \cos\theta = \frac{\sqrt{top\ fingers}}{2} \qquad \tan\theta = \frac{\sqrt{bottom\ fingers}}{\sqrt{top\ fingers}}$$

"Bottom fingers" refer to how many fingers are ***"below"*** the finger you've held down. ***"Top fingers"*** refer to how many fingers ***"above"*** the finger you've held down. Your thumb counts.

Step – 6 : Calculate the values for your trig expression using the appropriate formula.

For example: When you hold down your ring finger, there is *1* finger below your ring finder (your pinkie), and there are *3* fingers above your ring finger (your thumb, your index finger, and your middle finger). Therefore, Cos (π/6) = $\sqrt{3}$ /2 If you need Sin (π/6) = $\sqrt{1}/2$ = ½

1 Mark Questions

Q.1 Define Identity.

Q.2 Sin θ = $\frac{3}{4}$ for any value of θ (T/F)

Q.3 What is the value of ($1 - Cos^2θ$) $Cosec^2$ θ

Q.4 The value of $\frac{2 \tan 30^0}{1 - \tan^2 30^0}$ is

a) Sin 60^0 b) Cos 60^0 c) Tan 60^0 d) Sin 30^0

Q.5 In Triangle PQR right angled at Q. PQ+QR=25 cm PQ=5cm then the value of Sin P is

a) $\frac{7}{25}$ b) $\frac{24}{25}$ c) $\frac{12}{13}$ d) None of the these

Q.6 What is the Maximum value of $\frac{1}{\operatorname{cosec}\theta}$

Q.7 The value of Cos1^0 Cos 2^0 Cos3^0– Cos 180^0

a) 1 b) 0 c)- 1 d) None of these

Q.8 If $Cosec^2θ$ (1+Cos θ) (1- Cos θ) = K then value of K is

a) -1 b) 2 c) 1 d) -1

Q.9 If x tan 45^0 Cos 60^0 = Sin 60^0 Cot 60^0 then x is equal to

a) 1 b) $\sqrt{3}$ c) ½ d) 1/ $\sqrt{2}$

Q.10 ……… is The value of θ for which $\sqrt{3}$ Sin θ = Cos θ

Q.11 If tan A = ¾ and A+B =90^0 the value of Cot B is

a) ¾ b) 5/4 c) 3/5 d) ¾

Q12 If tan θ = ¾ then Cos^2 θ – $Sin^2θ$ =

a) 7/25 b) 1 c) -7/25 d) 4/25

Q13 If A, B and C are interior angles of Tringale ABC then Sin ($\frac{B+C}{2}$) =

a) Sin $\frac{A}{2}$ b) Cos $\frac{A}{2}$ c) Sin $\frac{A}{2}$ b) Cos $\frac{A}{2}$

Q14 The value of Sin A Cos $(90^0 – A)$ + Cos A Sin $(90^0 –A)$ is

a) -1 b) 0 c) 2 d) 4

Q.15 Define Angle.

ANSWERS

Q.1	Definition	Q.2	(F)	Q.3	(1)	Q.4	(c)	Q.5	(c)
Q.6	(1)	Q.7	(b)	Q.8	(c)	Q.9	(a)	Q.10	($30^{0)}$
Q.11	(d)	Q.12	(a)	Q.13	(b)	Q.14	(d)	Q.15	Definition

2 Mark Questions

Q.1 Evaluate:

$$\frac{2\ \text{Tan}\ 53^0}{\text{Cot}\ 37^0} - \frac{\text{Cot}\ 80^0}{\text{Tan}\ 10^0}$$

Q.2 Prove that:

$$(1- \text{Cos}^2\theta)\ \text{Cosec}^2\theta = 1$$

Q.3 Prove that:

$$\text{Cosec}\ \theta \sqrt{1 - \text{Cos}^2\theta} = 1$$

Q.4 Find the value of x, if

$$\text{Tan}\ 3x = \text{Sin}\ 45^0\ \text{Cos}\ 45^0 + \text{Sin}\ 30^0$$

Q.5 Solve the equation when $0^0 < \theta < 90^0$

$$3\ \text{Tan}^2\theta - 1 = 0$$

Q.6 Evaluate:

$$\text{Cos}^2 13^0 - \text{Sin}^2 77^0$$

Q.7 Evaluate:

$$\text{Sin}\ 60^0\ \text{Cos}\ 30^0 + \text{Cos}\ 60^0\ \text{Sin}\ 30^0$$

(Use Sin A Cos B + Cos A Sin B = Sin (A + B))

Q.8 If triangle ABC is a right angled at "C", then what is

Cos (A + B) + Sin (A + B) equal to

Hint *($A + B + C = 180^0 \Rightarrow A + B = 180^0 - C \Rightarrow A + B = 180^0 - 90^0 = 90^0$)*
i:e $A + B = 90^0$

Q.9 Evaluate:

$$\text{Tan}10^0\ \text{Tan}\ 15^0\ \text{Tan}\ 75^0\ \text{Tan}\ 80^0$$

Q.10 If A = 45^0, verify that

Sin 2 A = 2 Sin A Cos A

ANSWER

Q.1	(1)	Q.4	(15^0)	Q.5	($\theta = 30^0$)	Q.6	(0)
Q.7	(1)	Q.8	(1)	Q9	(1)		

3 Mark Questions

Q.1 If Cos A = $\frac{7}{25}$, find the value of Tan A + Cot A

Q.2 If θ is acute angle and Sin θ = Cos θ. Find the value of $2Tan^2\theta + Sin^2\theta - 1$.
(Hint: Sin θ = Cos θ ⟹ Sin θ / Cos θ = 1 ⟹ Tan θ = Tan 45^0, θ = 45^0)

Q.3 Given that Sin (A + B) = Sin A Cos B + Cos A Sin B. Find the value of 75^0
(Hint: Put A = 45^0, B = 30^0)

Q.4 Given that Sin 2 A = 2 Sin A Cos A. Find the value of Sin 120^0
(Hint: Put A = 60^0)

Q.5 Evaluate $4(Sin^4 60^0 + Cos^4 30^0) - 3(Tan^2 60 - Tan^2 45^0) + 5 Cos^2 45^0$

Q.6 Prove that Tan 1^0 Tan 2^0 Tan 3^0…….. Tan 89^0 = 1

Q.7 If Sin (θ + 36^0) = Cos θ, where θ +36^0 is acute angle. Find θ
(Hint: Use Cos θ = Sin (90 – θ)

Q.8 If Tan θ + Cot θ = 2. Find value of $Tan^2\theta + Cot^2\theta$. *(Hint: Squaring both sides*

Q.9 **Prove that:**

$$\frac{\sin\theta}{1-\cos\theta} = \cos ec\theta + \cot\theta$$

Hint L.H.S =
$$\frac{sin\theta}{1-cos\theta} = \frac{sin\theta}{1-cos\theta} \times \left(\frac{1+cos\theta}{1+cos\theta}\right)$$

Q.10 Express Sin 85^0 + Cosec 85^0 in terms of Trigonometric ratios of angles between 0^0 and 45^0. *(Hint: Use Sin (90 – θ) = Cos θ and Cosec (90 – θ) = Sec θ)*

ANSWERS

Q.1	$\frac{625}{168}$	Q.2	$\frac{3}{2}$	Q.3	$\frac{1}{\sqrt{2}}(\sqrt{3}+1)$	Q.4	$\frac{\sqrt{3}}{2}$	Q.5	1
Q.7	27^0	Q.8	2	Q.10	Cos 5^0 + Sec 5^0				

4 Mark Question

Q.1 In triangle ABC right angled at "C", if $\tan A = \frac{1}{\sqrt{3}}$. Find the value of $\sin A + \cos B + \cos A \sin B$

***(Alternate method: Hint**: $A + B + C = 180^0$, $A+B = 180^0 - C = 180^0 - 90^0 = 90^0$)*
$\sin A \cos B + \cos A \sin B = \sin(A + B) = \sin 90^0 = 1$

Q.2 If $\sin B = \frac{1}{2}$, Show that $3 \cos B - 4 \cos^3 B = 0$

Q.3 If $\tan \theta = \frac{20}{21}$ Show that

$$\frac{1 - \sin\theta + \cos\theta}{1 + \sin\theta + \cos\theta} = \frac{3}{7}$$

Q4 If $x = a \sec\theta + b \tan\theta$, $y = a \tan\theta + b \sec\theta$. Prove that $x^2 - y^2 = a^2 - b^2$

Q.5 Evaluate

$$\frac{\sin 18^\circ}{\cos 72^\circ} + \sqrt{3}(\tan 10^\circ \tan 30^\circ \tan 40^\circ \tan 50^\circ \tan 80^\circ)$$

Q.6 Prove the identity

$$\frac{\sin\theta}{(\cot\theta + \operatorname{cosec}\theta)} = 2 + \frac{\sin\theta}{(\cot\theta - \operatorname{cosec}\theta)}$$

Hint $$\frac{\sin\theta}{(\cot\theta + \operatorname{cosec}\theta)} - \frac{\sin\theta}{(\cot\theta - \operatorname{cosec}\theta)} = 2$$

Q.7 If $\sin\theta + \cos\theta = P$ and $\sec\theta + \operatorname{cosec}\theta = q$. Show that $q(P^2-1) = 2P$

Q.8 If $\sin\theta + \sin^2\theta = 1$ prove that $\cos^2\theta + \cos^4\theta = 1$
(Hint: $\sin\theta = 1 - \sin^2\theta = \cos^2 = \sin\theta = \cos^2\theta$)
Now $\cos^2\theta + \cos^4\theta = \cos^2\theta + (\cos^2\theta)^2 = \cos^2\theta + \sin^2\theta = 1$

Q.9 If $x = a\,Cos^3\,\theta$, $y = b\,Sin^3\,\theta$. Prove that $(\frac{x}{a})^{2/3} + (\frac{y}{b})^{2/3} = 1$

Q.10 Prove that

$(1 - Sin\ \theta + Cos\ \theta)^2 = 2\,(1 + Cos\,\theta)\,(1 - Sin\,\theta)$

(Hint $(1 - Sin\,\theta + cos\,\theta)^2$ ***use*** $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2\,ca$

$= 1 + (-Sin\ \theta)^2 + Cos^2\ \theta - 2\,Sin\theta - 2\,Sin\theta\,Cos\theta + 2\ Cos\theta$

$= 2 - 2\,Sin\ \theta + 2\,Cos\ \theta\ (1 - Sin\ \theta)$

$= 2(1 - Sin\ \theta) + 2\,Cos\ \theta(1 - Sin\ \theta) = 2\,(1 - Sin\theta)\,(1 + Cos\theta)$

Q.11 **Evaluate**

$$\frac{Sin^2\,20^0 + Cos^2 70^0}{Cos^2\,20^0 + Cos^2\,70^0} + \frac{Sin\,(90 - \theta)\,Sin\,\theta}{Tan\,\theta} + \frac{Cos\,(90 - \theta)\,Cos\,\theta}{Cot\,\theta}$$

Q.12 **Evaluate**

$$\frac{2}{3}(Cos^4 30^0 - Sin^4\,45^0) - 3(Sin^2 60 - Sec^2\,45^0) + \frac{1}{4}\ Cost^2 30^0$$

ANSWERS

Q.5 (2) Q11 (2) Q.12 $\frac{113}{24}$