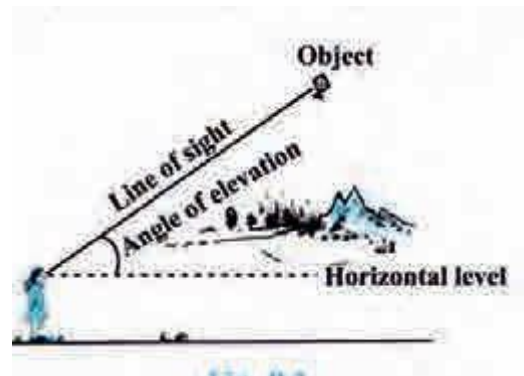
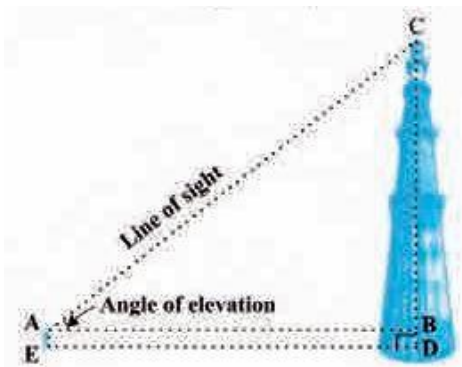


SOME APPLICATIONS TO TRIGONOMETRY

IMPORTANT FORMULAS AND CONCEPTS

Angle of Elevation:

In the below figure, the line AC is drawn from the eye of the student to the top of the minar is called the line of sight. The student is looking at the top of the minar. The angle BAC, so formed by the line of sight with the horizontal is called the angle of elevation of the top of the minar from the eye of the student. Thus, the line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.



Angle of Depression:

In the below figure, the girl sitting on the balcony is looking down at a flower pot placed on a stair of the temple. In this case, the line of sight is below the horizontal level. The angle so formed by the line of sight with horizontal is called the angle of depression. Thus the angle of depression of a point on the object being viewed is the angle formed by the line of sight with the horizontal when point is below the horizontal level, i.e. the case when we lower our head to look at the point being viewed.

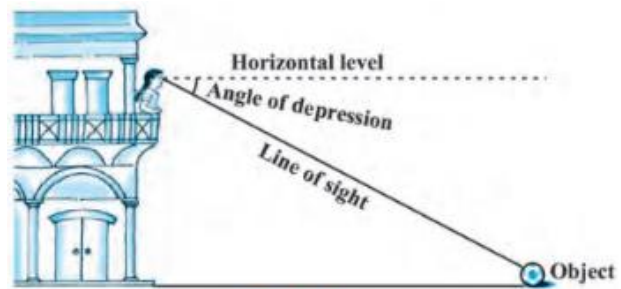
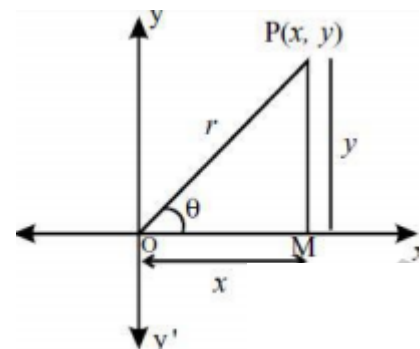


Fig. 9.3

Trigonometric Ratio (T – Ratio) of an acute angle of a Right Triangle:

In “XOY” – plane, let a revolving line “OP” starting from “OX”, traces angle $XOP = \theta$. From “P (x, y)” draw “PM perpendicular () to “OX”.

In right angled triangle OMP, OM = “x” (adjacent side), PM = “y” (opposite side): OP = “r” (hypotenuse).



1. $\sin \theta = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{y}{r}$
2. $\cos \theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{x}{r}$
3. $\tan \theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{y}{x}$
4. $\text{Cosec } \theta = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \frac{r}{y}$
5. $\text{Sec } \theta = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{r}{x}$
6. $\text{Cot } \theta = \frac{\text{Adjacent side}}{\text{Opposite side}} = \frac{x}{y}$

Reciprocal Relations:

1	$\sin A = \frac{1}{\text{cosec} A}$	$\text{cosec} A = \frac{1}{\sin A}$
2	$\tan A = \frac{1}{\cot A}$	$\cot A = \frac{1}{\tan A}$
3	$\cos A = \frac{1}{\sec A}$	$\sec A = \frac{1}{\cos A}$

Quotient Relations

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \text{and} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Trigonometric Ratios of Complementary Angles:

$$\begin{aligned} \Rightarrow \sin (90 - \theta) &= \cos \theta & \cos (90 - \theta) &= \sin \theta \\ \Rightarrow \tan (90 - \theta) &= \cot \theta & \cot (90 - \theta) &= \tan \theta \\ \Rightarrow \sec (90 - \theta) &= \text{cosec } \theta & \text{cosec } (90 - \theta) &= \sec \theta \end{aligned}$$

**TRIGONOMETRIC RATIOS FOR ANGLE OF MEASURE
0°, 30°, 45°, 60°, AND 90° IN TABULAR FORM**

$\theta =$	0°	30°	45°	60°	90°
$\sin \theta =$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta =$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta =$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined
$\operatorname{cosec} \theta =$	Undefined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec \theta =$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Undefined
$\cot \theta =$	Undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

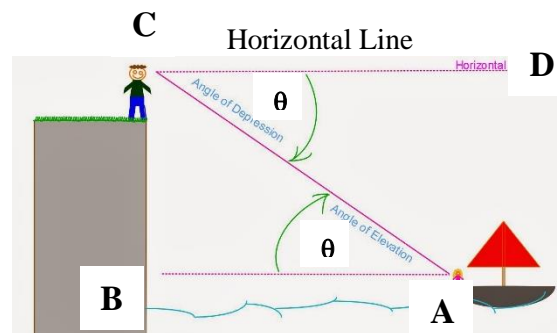
Remarks

In the figure above the angle of depression is angle $ACD = \theta$

Since $DC \parallel AB$ and AC is transversal.

Hence angle $ACD = \text{angle } CAB = \theta$

Thus angle $CAB = \theta$ is also be taken as angle of depression.



OBJECTIVE TYPE

(1 Marks Questions)

- Q.1** What is the angle of elevation of the sun when the shadow of a pole is $\sqrt{3}$ times the length of the pole?
- a) 30° b) 45° c) 60° d) None of these.
- Q.2** The shadow of a tower is 15 m, when the Sun's elevation is 30° . What is the length of the shadow, when the Sun's elevation is 60° ?
- a) 3 m b) 4 m c) 5 m d) 6 m
- Q.3** What is the angle of elevation of the Sun, When the shadow of a pole of height "x" m is $\frac{x}{\sqrt{3}}$.
- a) 30° b) 45° c) 60° d) 75°
- Q.4** A vertical stick 12 m long casts a shadow 8 m long on the ground. At the same time, a tower casts a shadow of 40 m long on the ground. The height of the tower is
- a) 60 m b) 65 m c) 70 m d) 72 m
- Q.5** The tops of two poles of height 24 m and 36 m are connected by a wire. If the wire makes an angle of 60° with the horizontal, then the length of the wire
- a) $8\sqrt{3}$ m b) 8m c) $6\sqrt{3}$ m d) 6 m
- Q.6** The shadow of a tower standing on a level plane is found to be 50 m longer when the Sun's elevation is 30° . When it is 60° . Then what is the height of the tower?
- a) 25 m b) $25\sqrt{3}$ c) $\frac{25}{\sqrt{3}}$ m d) 30 m
- Q.7** The angle of elevation of the tip of a tower from a point on the ground is 45° . Moving 21 m directly towards the base of the tower, the angle of elevation changes to 60° . What is the height of the tower, to the nearest metre?
- a) 48 m b) 49 m c) 50 m d) 51 m
- Q.8** The angle of depression from the top of a light house of two boats are 45° and 30° towards the west. If the two boats are 5 m apart, then the height of the light house is
- a) $(2.5\sqrt{3} - 1)$ m b) $2.5(\sqrt{3} - 1)$ m c) $2.5\sqrt{3} + 1)$ m d) $2.5(\sqrt{3} + 1)$ m

- Q.9** The angle of elevation of the top of an unfinished pillar at a point 150 m from the base is 30° . If the angle of elevation at the same point is to be 45° , then the pillar has to be raised to a height of how many meters?
- a) 59.4 m b) 61.4 c) 62.4 d) 63.4 m
- Q.10** From the top of a cliff 200 m high, the angles of depression of the top and bottom of a tower are observed to be 30° and 45° , respectively. What is the height of the tower?
- a) 400 m b) $400\sqrt{3}$ m c) $400/\sqrt{3}$ m d) None of these
- Q.11** On walking 120 m towards a chimney in a horizontal line through its base the angle of elevation of tip of the chimney changes from 30° to 45° . The height of the chimney is
- a) 120 m b) $60(\sqrt{3}m - 1)$ c) $60(\sqrt{3}m + 1)$ d) None of these
- Q.12** A man standing at a point "P" is watching the top of elevation of 30° . The man walks some distance towards the tower and then his angle of elevation of the top of the tower is 60° . If the height of the tower is 30 m, then the distance he moves is
- a) 20 m b) $20\sqrt{3}m$ c) 22 m d) $22\sqrt{3}m$
- Q.13** The angle of elevation of the top of a tower from the bottom of a building is twice that from its top. What is the height of the building, if the height of the tower is 75 m and the angle of elevation of the top of the tower from the bottom of the building is 60° ?
- a) 25 m b) 37.5 m c) 50 m d) 60 m
- Q.14** The angles of elevation of the top of a tower from two points which are at distance of 10 m and 5 m from the base of the tower and in the same straight line with it are complementary. The height of the tower is
- a) 5 m b) 15 m c) $\sqrt{15}m$ d) $\sqrt{75}m$
- Q.15** The angles of elevation of a top of an inaccessible tower from two points on the same straight line from the base of the tower are 30° and 60° , respectively. If the points are separated at a distance of 100 m, then the height of the tower is close to
- a) 86.6 m b) 84.6 m c) 82.6 m d) 80.6 m
- Q.16** Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m. What is the distance between their tops?
- a) 13 m b) 17 m c) 18 m d) 23 m

Directions: (Q No. 17 – 20) read the following information carefully to answer the questions that follow.

As seen from the top and bottom of a building of height “h m”, the angles of elevation of the top of a tower of height $\frac{(3+\sqrt{3})h}{2}$ m and “ α ” and “ β ”, respectively.

Q.17 If $\beta = 30^\circ$, then what is the value of $\tan \alpha$

- a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) None of these.

Q.18 If $\alpha = 30^\circ$, then what is the value of $\tan \beta$

- a) 1 b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) None of these

Q.19 If $\alpha = 30^\circ$ and h = 30 m, then what is the distance between the base of the building and the base of the tower

- a) $15 + 15\sqrt{3}m$ b) $30 + 15\sqrt{3}m$ c) $45 + 15\sqrt{3}m$ d) None of these

Q.20 If $\beta = 30^\circ$ and if q is the angle of depression of the foot of the tower as seen from the top of the building, then what is $\tan q$ equal to ?

- a) $\frac{3-\sqrt{3}}{3\sqrt{3}}$ b) $\frac{3+\sqrt{3}}{3\sqrt{3}}$ c) $\frac{2-\sqrt{3}}{3\sqrt{3}}$ d) None of these

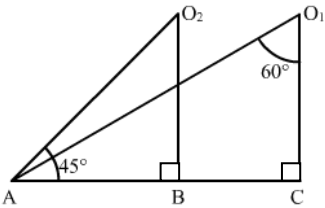
1 Mark Questions

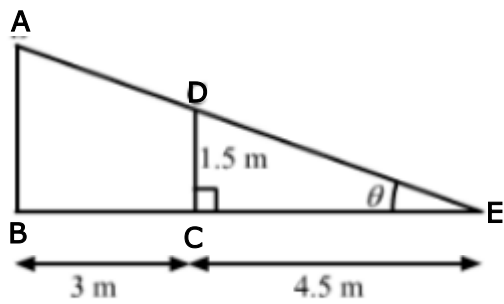
- Q.1 Define line of sight Q.2 Define angle of Elevation
Q.3 Define angle of Depression

ANSWERS

Q.1	(a)	Q.2	(c)	Q.3	(c)	Q.4	(a)	Q.5	(a)
Q.6	(b)	Q.7	(c)	Q.8	(d)	Q.9	(d)	Q.10	(d)
Q.11	(c)	Q.12	(b)	Q.13	(c)	Q.14	(c)	Q.15	(a)
Q.16	(a)	Q.17	(b)	Q.18	(a)	Q.19	(c)	Q.20	(a)

2 Marks Questions

- Q.1 The height of a tower is 10 m. What is the length of its shadow when sun's altitude is 45° ?
- Q.2 If the ratio of the height of a Tower and the length of its shadow is $\sqrt{3} : 1$. What is the angle of elevation of the sun?
- Q.3 What is the angle of elevation of the sun when the length of the shadow of a vertical pole is equal to its height?
- Q.4 A vertically straight tree 15 m high is broken by the wind in such a way that its top just touches the ground and makes an angle of 60° with the ground. At what height from the ground did the tree break?
- Q.5 An observer 1.5 m tall is 28.5 m away from a tower. The angle of elevation of the top of a tower from her eyes is 45° . What is the height of a tower?
- Q.6 In the given figure what are the angles of depression from the observing position O_1 and O_2 of the object at "A"?
- 
- Q.7 A ladder makes an angle of 60° with the ground when placed against wall. If the foot of ladder is 2 m away from the wall, then length of the ladder is.
- Q.8 The length of shadow of tower on the plane ground is $\sqrt{3}$ times the height of tower. The angle of elevation of sun is?
- Q.9 The angle of depression of a car standing on the ground from the top of a 75 m tower is 30° . Then the distance of car from the base of the tower is?
- Q.10 A kite is flying at a height of 60 m above the ground. The inclination of the string with the ground is 60° . Find the length of string, assuming that there is no slack in the string.
- Q.11 A tower is 50 m high. Its shadow is "x" meters shorter, when the sun's altitude is 45° than when it is 30° . Find value of "x".



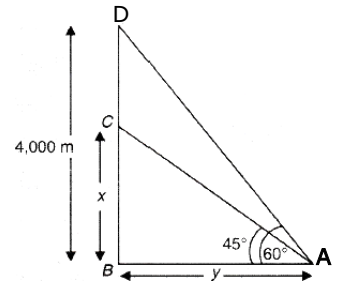
Q.12 If a 1.5 m tall girl stands at a distance of 3 m from a lamppost and casts shadow of length 4.5 m on the ground. Find the length /height of the lamppost. (Hint: In ΔDCE , $\tan \theta = \frac{1.5}{4.5} = \frac{1}{3}$ and in ΔABE $\tan \theta = \frac{AB}{7.5}$, $AB = \frac{7.5}{3} = 2.5$ m)

ANSWERS

Q.1	(10 m)	Q.2	(60°)	Q.3	(45°)	Q.4	(6.9 m)
Q.5	(30 m)	Q.6	($30^\circ, 45^\circ$)	Q.7	($2\sqrt{3}$)	Q.8	(30°)
Q.9	$75\sqrt{3}$	Q.10	($40\sqrt{3}$)	Q.11	($50\sqrt{3} - 1$)		

3 Mark Questions

- Q.1 An aeroplane when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant, when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplane at that instant.

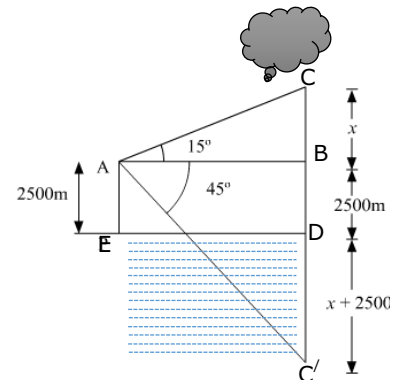


- Q.2 A tower stands vertically on the ground. From a point on the ground, 20 m away from the foot of the tower, the angle of elevation of the top of tower is 60° . What is the height of the tower?
- Q.3 A ladder is placed along a wall of a house such that its upper end is touching the top of the wall. The foot of the ladder is 2 m away from the wall and the ladder is making an angle of 60° with the level of ground. Determine the height of the wall.
- Q.4 The length of a string between a kite and a point on the ground is 90 m. If the string makes an angle θ with the ground level such that $\tan \theta = \frac{15}{8}$. How high is the kite?
- Q.5 A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff. At a point on the plane 70 meters away from the tower, an observer notices that the angles of elevation of the top and bottom of the flag staff are respectively 60° and 45° . Find the height of flag staff and that of tower.
- Q.6 The shadow of a tower when the angle of elevation of the sun is 45° is found to be 10 m longer than when it was 60° . Find the height of tower.
- Q.7 The angles of elevation of the top of a rock from the top and foot of a 100 m high tower are respectively 30° and 45° . Find the height of the rock.
- Q.8 As observed from the top of a 15 m tall light house, the angles of depression of two ships approaching it are 30° and 45° . If one ship is directly behind the other. Find the distance between the two ships.
- Q.9 The angles of depression of two ships from the top of a light house and on the same side of it are found to be 45° and 30° respectively. Find the height of the light house.

Q.10 The length of a shadow of a tower standing on level plane is found to be 2 x meters longer, when the sun's altitude is 30° than when it was 45° . Prove that the height of tower is $x(\sqrt{3} + 1)$.

Q.11 The angle of elevation of a tower from a point on the same level as the foot of the tower is 30° . On advancing 150 m towards the foot of the tower, the angle of the elevation of the tower becomes 60° . Show that the height of the tower is 129.9 m (use $\sqrt{3} = 1.732$).

Q.12 The angle of elevation of a stationary cloud from a point 2500 m above the lake is 15° and the angle of depression of its reflection in the lake is 45° . What is the height of the cloud above the lake level? (Use $\tan 15^\circ = 0.268$).



(Hint let "C'" be the reflection of cloud "C" in the lake. Then "C'D" = 2500 m + x, so that $BC' = 5000 + x$.)

In triangle ABC' $\tan 45^\circ = \frac{BC'}{AB}$

➤ $AB = 5000 + x$

Take ΔABC , $\tan 15^\circ = \frac{BC}{AB} = \frac{x}{5000+x}$ and solve it.

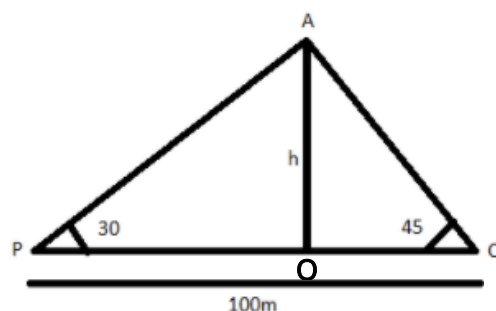
ANSWERS

Q.1	$\frac{4000(\sqrt{3}-1)}{\sqrt{3}}$	Q.2	$20\sqrt{3}$	Q.3	$2\sqrt{3}$	Q.4	79.41
Q.5	51.24 m, 70 m	Q.6	23.66 m	Q.7	236.5 m	Q.8	109.5 m
Q.9	273.2 m	Q.12	$2500\sqrt{3}$ m				

4 MARK QUESTIONS

- Q.1 The horizontal distance between two towers is 140 m. The angle of elevation of the top of the first tower when seen from the top of the second tower is 30° . If the height of the second tower is 60 m. Find the height of the first tower.
- Q.2 The angle of elevation of a cloud from a point 60 m above the lake is 30° and angle of depression of the reflection of cloud in the lake is 60° . Find the height of the cloud.

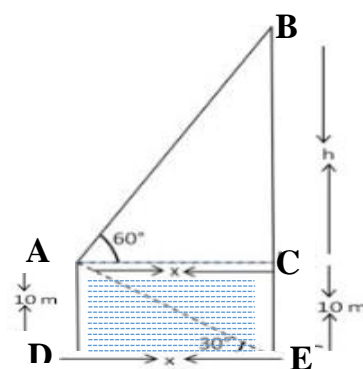
- Q.3 There is a small island in the middle of a 100 m wide river and a tall tree stands on the island. "P" and "Q" are points directly opposite to each other in the two banks and in a line with a tree. If the angles of elevation of the top of tree from "P" and "Q" are respectively 30° and 45° . Find the height of the tree.



$$(Hint: \tan 30^\circ = \frac{OA}{OP}, \tan 45^\circ = \frac{OA}{OQ}, OP + OQ = (\sqrt{3}+1)h, h = \frac{100}{\sqrt{3}+1})$$

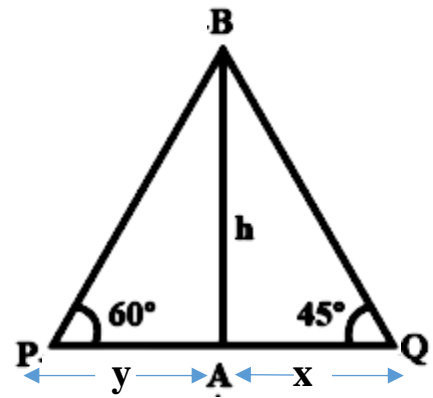
- Q.4 From the top of a building 60 m high the angles of depression of the top and the bottom of a tower are observed to be 30° and 60° . Find the height of the tower.
- Q.5 A man standing on the deck of a ship which is 10 m above water level. He observe the angle of depression of the base of the hill as 30° . Calculate the distance of the hill from the ship and the height of the hill.

$$(Hint: \tan 60^\circ = \frac{\sqrt{h}}{x}, h = \sqrt{3}, \tan 30^\circ = \frac{10}{x}, x = 10\sqrt{3}, h = \sqrt{3}(10\sqrt{3} = 30).$$



- Q.6 As observed from the top of a light house 100 m above the sea level, the angle of depression of a ship, sailing directly towards it changes from 30° to 45° . Determine the distance travelled by ship.
- Q.7 An aeroplane at a altitude of 200 m observes the angles of depression of opposite points on the two banks of a river to be 45° and 60° . Find the width of the river.
- Q.8 The shadow of a vertical tower on the level ground increases by 10 m, when the altitude of the sun changes from angle of elevation 45° to 30° . Find the height of the tower.

- Q.9 A fire in a building “B” is reported on telephone to two fire stations “P” and “Q” 20 Km apart from each other on a straight road. “P” observes that the fire is at an angle of 60° to the road and “Q” observes that it is at an angle of 45° to the road. Which station should send its team and how much will this team have to travel.



(Hint: $AP < AQ$, station “P” must send the team.

$$\tan 60^\circ = \frac{h}{y}, h = \sqrt{3}y, \tan 45^\circ = \frac{h}{x}, h = x, x + y = 20 \text{ proceed)}$$

- Q.10 From a window 15 m high above the ground in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are 30° and 45° respectively. Show that the height of opposite house is 23.66 m (take $\sqrt{3} = 1.732$).

ANSWER

Q.1	140.83 mtrs.	Q.2	120 m	Q3	$\frac{100}{\sqrt{3}+1}$	Q4	40 m
Q.5	$10\sqrt{3}, 40$	Q.6	73.2 m.	Q7	$200 \left(\frac{\sqrt{3}+1}{\sqrt{3}} \right)$	Q.8	$5(\sqrt{3} + 1)$
Q.9	7.32 Km	Q.10	23.66 m				